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New possibilities in the DECAY0/GENBB code: $4\beta0\nu$ decay and $2\beta2\nu$ decay with Lorentz violation

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Abstract

Two new possibilities were added recently to the DECAY0/GENBB event generator to generate initial kinematics of electrons emitted in: (1) neutrinoless quadruple β decay, (2) Lorentz violated two-neutrino double β decay. Formulae used in the code are given together with comparison of generated energy spectra with theoretical distributions.

1 Neutrinoless quadruple β decay

Neutrinoless quadruple β decay $(A, Z) \to (A, Z \pm 4)$ was considered at the first time in [1] which still is the only paper on this subject. Four-dimensional energy distribution for the emitted $\beta^{\pm}$ particles is equal [2]:

$$\rho_{1234}(t_1, t_2, t_3, t_4) = e_1 p_1 F(t_1, Z) e_2 p_2 F(t_2, Z) e_3 p_3 F(t_3, Z) e_4 p_4 F(t_4, Z), \tag{1}$$

where t_i is the kinetic energy of the i-th e^- or e^+ (all energies here are in units of the electron mass m_0c^2), $e_i = t_i + 1$ is the total energy of i-th particle, p_i is its momentum $p_i = \sqrt{t_i(t_i + 2)}$ (in units of m_0c), and $F(t, Z)$ is the Fermi function which takes into account the influence of the electric field of the nucleus on the emitted electrons or positrons. The energy of the 4-th particle, because of energy conservation, is just calculated as $t_4 = t_0 - t_1 - t_2 - t_3$, where t_0 is the energy available in the 4β process.

The Fermi function is defined in the DECAY0/GENBB code [3] as:

$$F(t, Z) = const \cdot p^{2\gamma-2} \exp(\pi s) \mid \Gamma(\gamma + is) \mid^2, \tag{2}$$

where $\gamma = \sqrt{1 - (\alpha Z)^2}$, $s = \alpha Ze/p$, $\alpha = 1/137.036$ is the fine structure constant, Z is the atomic number of the daughter nucleus ($Z > 0$ for β^- and $Z < 0$ for β^+ decay) and Γ is the gamma function.

Energy distribution for single electron can be found as:

$$\rho_1(t_1) = e_1 p_1 F(t_1, Z) \int_0^{t_0-t_1} e_2 p_2 F(t_2, Z) dt_2 \int_0^{t_0-t_1-t_2} e_3 p_3 F(t_3, Z) e_4 p_4 F(t_4, Z) dt_3. \tag{3}$$

In case of the Primakoff-Rosen (PR) approximation [4] $F(t,Z) \sim e/p$ (adequate only for $Z > 0$, i.e. for β^-, $2\beta^-$ and $4\beta^-$ decays), it is possible to take the integral (3) analytically. This gives:

$$\begin{aligned} \rho_1^{PR}(t_1) &= (t_1+1)^2(t_0-t_1)^2[(t_0-t_1)^6+24(t_0-t_1)^5+252(t_0-t_1)^4+ \\ &+ 1344(t_0-t_1)^3+3780(t_0-t_1)^2+5040(t_0-t_1)+2520]. \end{aligned} \tag{4}$$

Energy spectra for single electron calculated numerically without the PR approximation (Eq. (3)) and analytically with the PR approximation (Eq. (4)) are shown in Fig. 1(top) for $4\beta0\nu$ decay ${}^{150}_{60}\mathrm{Nd} \to {}^{150}_{64}\mathrm{Gd}$ with $Q_{4\beta} = 2085(6)$ keV [5]. One can see that the PR approximation is quite good for $4\beta^-$ decay (as it is usually good for $2\beta^-$) with noticeable difference only at energies < 200 keV.

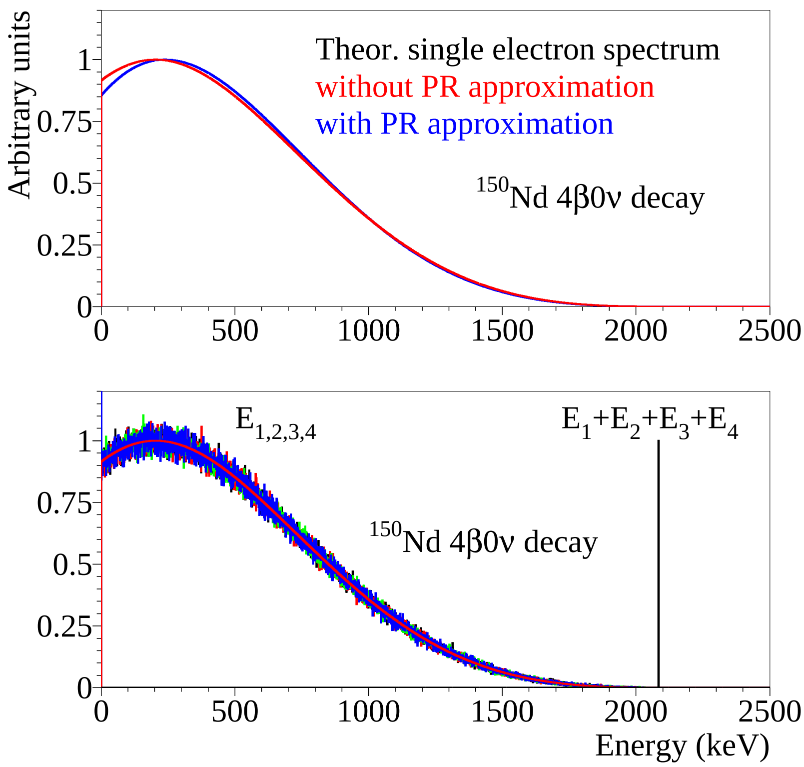

Figure 1: Top: Theoretical single electron energy distributions for $4\beta0\nu$ decay ^{150}Nd $\to$ ^{150}Gd with $Q_{4\beta} = 2085$ keV with and without the PR approximation. Bottom: Energy spectra (total and single electron for four electrons in different colors) of the generated ^{150}Nd $4\beta0\nu$ events in comparison with theoretical distribution without the PR approximation.

Energy spectra of individual electrons in ^{150}Nd $4\beta0\nu$ decay generated with the DECAY0/GENBB are shown in Fig. 1(bottom) in different colours; they have the same shape (as it should be) and are in agreement with the theoretical distribution without the PR approximation. Directions of the emitted β particles currently are sampled isotropically because angular distributions are "too messy" [6].

It is interesting to note that among 288 long-lived or stable isotopes that are present in the naturally occuring isotope mixture of elements [7], there are only 3 candidates for $4\beta^-$ decay (^{96}Zr, ^{136}Xe, ^{150}Nd) and 2 candidates for 4EC (^{124}Xe, ^{130}Ba). All of them are listed in [1] where also 2 other isotopes (^{148}Gd with $T_{1/2} = 75$ y, and ^{154}Dy with $T_{1/2} = 3 \times 10^6$ y) are given which, however, are not present in the natural isotope mixture due to their short half-lives.

2 Lorentz violated two-neutrino double β decay

According to the Lorentz invariance (LI) principle, any physical law should keep its form invariant in any inertial frame. LI is one of the founding principles of modern physics, but it could be only approximate symmetry of our local space-time possibly modified at some scale outside of our experience. As any fundamental principle, LI should be checked with the highest available to-date sensitivity; see reviews [8] on searches for LI violation. As was noted in [9, 10], LI could be tested also in studies of double beta decay as LI violation leads to energy spectra of emitted particles different from those in usual $2\beta2\nu$ process.

Three-dimensional energy and angular distribution for the emitted $\beta^{\pm}$ particles is equal [11]:

$$\rho_{12}(t_1, t_2, \cos\theta) = e_1 p_1 F(t_1, Z) e_2 p_2 F(t_2, Z)(t_0 - t_1 - t_2)^4 (1 - \beta_1 \beta_2 \cos\theta), \tag{5}$$

where β_i is velocity of i-th electron or positron in units of c: $\beta_i = v_i/c = p_i/e_i$, and θ is the angle between particles' directions.

Energy distribution for single electron can be found as:

$$\rho_1(t_1) = e_1 p_1 F(t_1, Z) \int_0^{t_0 - t_1} e_2 p_2 F(t_2, Z)(t_0 - t_1 - t_2)^4 dt_2. \tag{6}$$

Distribution for sum of electron's energies $t = t_1 + t_2$ is:

$$\rho_{1+2}(t) = \int_0^t e_1 p_1 F(t_1, Z) e_2 p_2 F(t_2, Z)(t_0 - t_1 - t_2)^4 dt_1, \tag{7}$$

where $t_2 = t - t_1$.

The Primakoff-Rosen approximation $F(t, Z) \sim e/p$ gives possibility to calculate the integrals analytically as:

$$\rho_1^{PR}(t_1) = (t_1 + 1)^2 (t_0 - t_1)^5 [(t_0 - t_1)^2 + 7(t_0 - t_1) + 21], \tag{8}$$

$$\rho_{1+2}^{PR}(t) = t(t_0 - t)^4 (t^4 + 10t^3 + 40t^2 + 60t + 30). \tag{9}$$

The last expression is given also in [10], Eq. (4).

Distribution for sum of energies of electrons in Lorentz violated (LV) $2\beta2\nu$ decay for ^{100}Mo $\rightarrow$ ^{100}Ru with $Q_{2\beta} = 3034$ keV is given in Fig. 2 in comparison with spectra in other 2β modes: neutrinoless $2\beta0\nu$, neutrinoless with emission of Majoron(s) $2\beta0\nu$Mn with different spectral indexes SI $= n$, and two-neutrino $2\beta2\nu$. In the PR approximation, formulae for different Majorons are similar to Eq. (9):

$$\rho_{1+2}^{PR}(t) = t(t_0 - t)^n (t^4 + 10t^3 + 40t^2 + 60t + 30), \tag{10}$$

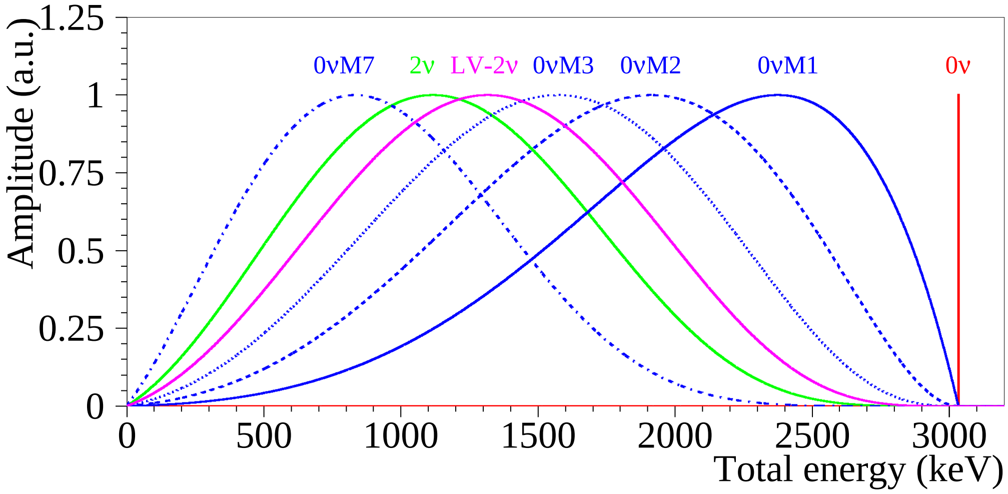

Figure 2: Theoretical total electron energy distributions for 2β decay ^{100}Mo $\rightarrow$ ^{100}Ru with $Q_{2\beta} = 3034$ keV for different 2β modes: neutrinoless $2\beta0\nu$, neutrinoless with emission of Majoron(s) $2\beta0\nu$Mn with different spectral indexes $n = 1, 2, 3, 7$, two-neutrino $2\beta2\nu$, and Lorentz-violated LV-$2\beta2\nu$.

with $n = 1, 2, 3, 7$. Thus, LV-$2\beta2\nu$ decay is equivalent to Majoron with the spectral index $n = 4$ (which, hovever, was not proposed by theorists in the literature to-date), and usual $2\beta2\nu$ decay corresponds to $n = 5$.

Fig. 3(top) shows single electron and total electrons energy spectra for ^{100}Mo LV-$2\beta2\nu$ decay without (Eq. (6,7)) and with (Eq. (8,9)) the Primakoff-Rosen approximation. One can see that numerically calculated spectra without PR approximation are very close to those obtained analytically with the PR approximation. Energy spectra of electrons generated with the DECAY0/GENBB are shown in Fig. 3(bottom) in different colours; they have the same shape (as it should be) and are in agreement with the theoretical distribution without the PR approximation.

References

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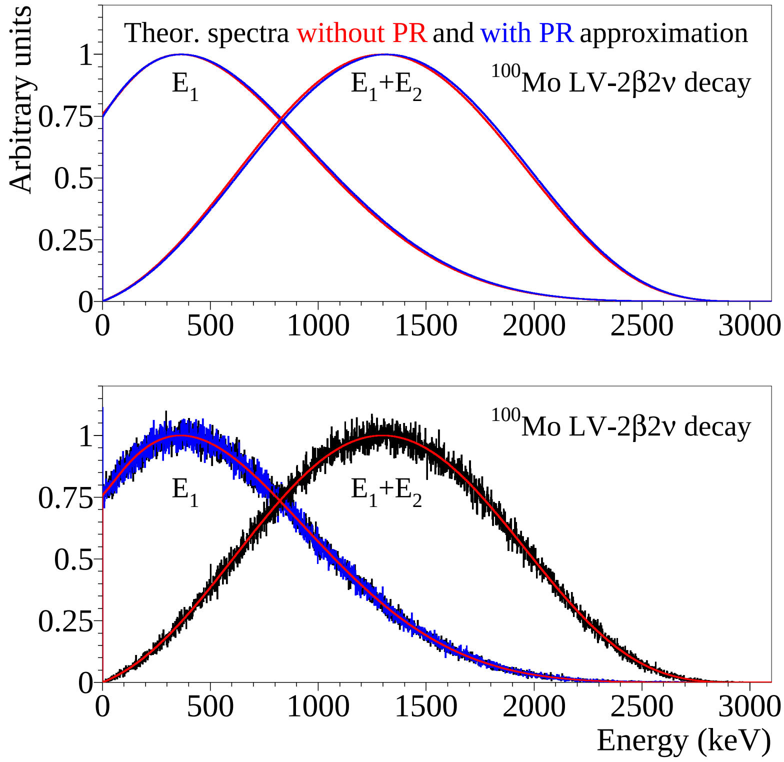

Figure 3: Top: Theoretical single and total electron energy distributions for LV-$2\beta2\nu$ decay ^{100}Mo $\rightarrow$ ^{100}Ru with $Q_{2\beta}$ = 3034 keV with and without the PR approximation. Bottom: Energy spectra (single and total for two electrons in different colors) of the generated ^{100}Mo LV-$2\beta2\nu$ events in comparison with theoretical distributions without the PR approximation.

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