NEMO note
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# Energy and angular distributions of electrons in $2\beta0\nu$ decay due to right-handed currents

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### Abstract

Formulae are presented which are used in the DECAY0/GENBB event generator for generation of events in $2\beta0\nu$ decay due to right-handed currents. Distributions are shown for all $2\beta$ isotopes currently in use in the NEMO-3 set-up.

## 1 Formulae for energy and angular distributions

In that follows, mainly the paper of Doi et al., 1988 [1] will be used, where approximations for energy and angular distributions of electrons emitted in neutrinoless $2\beta$ decay were considered. It summarizes and simplifies earlier works of the same authors [2, 3, 4] (see also [5, 6, 7]).

Sampling of energies and angles of $e^-$ or $e^+$ in $2\beta0\nu$ decay is based on 2-dimensional distribution $\rho_{1\theta}(t_1, \cos\theta)$ from which 1-dimensional distribution $\rho_1(t_1)$ is calculated [8]:

$$\rho_1(t_1) = \int_0^\pi \rho_{1\theta}(t_1, \cos\theta)\, d(\cos\theta). \tag{1}$$

Here $t_i$ is the kinetic energy of the $i$-th $e^-$ or $e^+$, and $\theta$ is the angle between the particle directions. The energy of the first $e^-$ or $e^+$ is sampled in accordance with $\rho_1(t_1)$. The energy of the second particle, because of energy conservation, is just calculated as $t_2 = t_0 - t_1$, where $t_0$ is the energy available in the $2\beta$ process (all energies here are in units of the electron mass $m_0c^2$). Finally the angle $\theta$ is sampled from $\rho_{1\theta}(t_1, \cos\theta)$ with fixed $t_i$, supposing isotropic emission for the first particle [8].

The momentum of the $i$-th electron, $p_i$, which appears in the formulae below, is given by $p_i = \sqrt{t_i(t_i+2)}$ (in units of $m_0c$) and its velocity, $\beta_i$, by $\beta_i = p_i/e_i$ (in units of $c$) where $e_i = t_i + 1$ is total energy of $i$-th particle. The Fermi function is defined as

$$F(t, Z) = const \cdot p^{2\gamma-2} \exp(\pi s) \mid \Gamma(\gamma + is) \mid^2, \tag{2}$$

where $\gamma = \sqrt{1-(\alpha Z)^2}$, $s = \alpha Ze/p$, $\alpha = 1/137.036$ is fine structure constant, $Z$ is the atomic number of the daughter nucleus ($Z > 0$ for $\beta^-$ and $Z < 0$ for $\beta^+$ decay) and $\Gamma$ the gamma function[1].

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[1] In the Primakoff-Rosen approximation $F(t, Z) \sim e/p$, which is adequate only for $Z > 0$ ($\beta^-$ and $2\beta^-$ decays).

In accordance with [1], for $2\beta0\nu$ decay and $0^+ - 0^+$ transition $\rho_{1\theta}(t_1, \cos\theta)$ is equal:

$$\rho_{1\theta}(t_1, \cos\theta) = e_1 p_1 F(t_1, Z) e_2 p_2 F(t_2, Z)(A(t_1) + B(t_1) \cdot \beta_1 \beta_2 \cos\theta), \tag{3}$$

where $A(t)$ and $B(t)$ depend on mechanism of $2\beta$ decay.

If to work only with one of three mechanisms of $2\beta0\nu$ decay (due to non-zero neutrino mass, or $\lambda$, or $\eta$ terms in right-handed currents) at one time independently (as it is implemented in the GENBB/DECAY0 event generator), $A(t)$ and $B(t)$ will be given below; working with all three mechanisms simultaneously, $A(t)$ and $B(t)$ will be more complex including different interference terms, see [1].

(1) For neutrino mass mechanism, $A_m$ and $B_m$ don't depend on electron energy and are equal:

$$A_m(t_1) = 1, \quad B_m(t_1) = -1. \tag{4}$$

It gives simple distribution:

$$\rho_{1\theta}(t_1, \cos\theta) = e_1 p_1 F(t_1, Z) e_2 p_2 F(t_2, Z)(1 - \beta_1 \beta_2 \cos\theta), \tag{5}$$

which is used in the GENBB/DECAY0 code already many years [8].

(2) For mechanism related with the $\lambda$ term in right-handed currents, $A_\lambda$ and $B_\lambda$ are sum of products of some functions of electron energy and nuclear matrix elements:

$$A_\lambda(t_1) = A_1 \chi_{2-}^2 + A_2 \chi_{2-} \chi_{1+} + A_3 \chi_{1+}^2, \tag{6}$$

$$B_\lambda(t_1) = \frac{1}{2}(e_1 - e_2)^2 \chi_{2-}^2 - \frac{4}{81} \chi_{1+}^2. \tag{7}$$

Here $A_i$ are functions of electron energy determined as:

$$A_1 = \frac{1}{2} \frac{(e_1 e_2 - 1)(e_1 - e_2)^2}{e_1 e_2}, \tag{8}$$

$$A_2 = -\frac{2}{9} \frac{(e_1 - e_2)^2}{e_1 e_2}, \tag{9}$$

$$A_3 = \frac{2}{81} \frac{e_1 e_2 - 1}{e_1 e_2}. \tag{10}$$

Terms $\chi_{1+}$ and $\chi_{2-}$ are combinations of ratios $\chi_\alpha = M_\alpha / M_{GT}$ of different nuclear matrix elements (NME) $M_\alpha$ to Gamow-Teller NME $M_{GT}$:

$$\chi_{1\pm} = \chi'_{GT} \pm 3\chi'_F - 6\chi'_T, \tag{11}$$

$$\chi_{2\pm} = \chi_{GT\omega} \pm \chi_{F\omega} - \frac{1}{9} \chi_{1\mp}. \tag{12}$$

In the last equation $\chi_{1\mp}$ was written instead of $\chi_{1\pm}$ in formula (3.5.16) in Ref. [4], in accordance with further correction (see footnote on page 146 in Ref. [7]).

The ratios $\chi_\alpha = M_\alpha/M_{GT}$ of different NMEs $M_\alpha$ to the Gamow-Teller NME $M_{GT}$ are defined in Eqs. (3.5.2 – 3.5.9) of Ref. [4][2]. All these 6 NMEs should be calculated for each nucleus of interest before to be used in Eqs. (6) and (7).

Such a situation is slightly inconvenient: (1) if the $\chi_i$ values were not calculated in some theoretical works for specific nucleus, you cannot calculate the $\rho_{1\theta}(t_1, \cos\theta)$ distribution (and generate events of $2\beta0\nu$ decay); (2) values of NMEs calculated by different authors surely will be different and, thus, $\rho_{1\theta}(t_1, \cos\theta)$ also will be different.

However, in case of the $\lambda$ term in right-handed currents, it is possible to make further simplifications. In accordance with Ref. [4] (see page 69), the term $\chi_{2-}$ gives the main contribution. Further, approximating the $e_1e_2 - 1$ by $e_1e_2$ in Eq. (8) (i.e. neglecting by electron mass in comparison with beta particle total energies), we will obtain

$$A_\lambda = B_\lambda = \frac{1}{2}(e_1 - e_2)^2\chi_{2-}^2, \tag{13}$$

and

$$\rho_{1\theta}(t_1, \cos\theta) = e_1p_1F(t_1, Z)e_2p_2F(t_2, Z)(e_1 - e_2)^2(1 + \beta_1\beta_2\cos\theta). \tag{14}$$

This approximation was implemented in the GENBB/DECAY0 event generator also many years ago and is used up to date.

(3) For mechanism related with the $\eta$ term in right-handed currents, situation is more complex:

$$A_\eta(t_1) = A_1\chi_{2+}^2 + A_2\chi_{2+}\chi_{1-} + A_3\chi_{1-}^2 + A_4\chi_R'^2 + A_5\chi_R'\chi_P' + A_6\chi_P'^2, \tag{15}$$

$$B_\eta(t_1) = \frac{1}{2}(e_1 - e_2)^2\chi_{2+}^2 - \frac{4}{81}\chi_{1-}^2 + \frac{8}{r^2}(\frac{\zeta}{6}\chi_P' - \chi_R')^2 - \frac{8}{9}\chi_P'^2, \tag{16}$$

where additional functions $A_i$ are:

$$A_4 = \frac{8}{r^2}\frac{e_1e_2 + 1}{e_1e_2}, \tag{17}$$

$$A_5 = -\frac{8}{3r^2}\frac{1}{e_1e_2}(\zeta(e_1e_2 + 1) - 2re_0), \tag{18}$$

$$A_6 = \frac{2}{9r^2}\frac{1}{e_1e_2}[(\zeta^2 + 4r^2)(e_1e_2 + 1) - 4\zeta re_0]. \tag{19}$$

Here appear ratios $\chi_R', \chi_P'$ of two additional NMEs to the Gamow-Teller NME. Product $r = m_0c^2 \cdot R_A$ with $R_A = 1.2\sqrt[3]{A}$ fm is equal

$$r = 3.107526 \cdot 10^{-3}\sqrt[3]{A}, \tag{20}$$

and

$$\zeta = 3\alpha Z + re_0. \tag{21}$$

Once again, to calculate distribution $\rho_{1\theta}(t_1, \cos\theta)$, we should know 8 NMEs calculated for our nucleus of interest. More often (see Ref. [4], page 69) $\eta$ distribution is of "mountain" type (as for $m_\nu$ term, Eq. (5)), however sometimes cancellation between NMEs could result also in "valley" type distribution (as for $\lambda$ term, Eq. (14)). See examples on Fig. 6.7 of Ref. [4], where energy distribution of single electrons for $^{48}$Ca is of "valley" type while for $^{76}$Ge it is of "mountain" type. Angular distribution generally is of $1 + \beta_1\beta_2\cos\theta$ type, as for $\lambda$ term.

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[2] Sometimes notations $\chi_{Fq}, \chi_{GTq}$ are used instead of $\chi_F', \chi_{GT}'$, respectively.

## 2 Energy distributions for the NEMO-3 isotopes

Below we give values of the NME ratios $\chi_\alpha$ calculated in theoretical works [9, 6, 10] (where they were calculated for bigger set of $2\beta$ decaying nuclides) and draw corresponding energy distributions for $m_\nu, \lambda$ and $\eta$ terms for isotopes currently investigated in the NEMO-3 set-up: $^{48}$Ca, $^{82}$Se, $^{96}$Zr, $^{100}$Mo, $^{116}$Cd, $^{130}$Te, $^{150}$Nd (and also for $^{76}$Ge). In addition, the $\chi_\alpha$ values for different isotopes also can be found f.e. in: [3] ($^{76}$Ge, $^{130}$Te), [4] ($^{48}$Ca, $^{76}$Ge, $^{82}$Se, $^{128}$Te, $^{130}$Te), [5] ($^{48}$Ca, $^{76}$Ge, $^{82}$Se, $^{128}$Te, $^{130}$Te), [11] ($^{76}$Ge), [12] ($^{76}$Ge), [13] ($^{76}$Ge), [14] ($^{48}$Ca, $^{76}$Ge, $^{100}$Mo, $^{128}$Te, $^{130}$Te), [15] ($^{48}$Ca, $^{76}$Ge, $^{100}$Mo, $^{130}$Te, $^{136}$Xe).

Table 1: Ratios $\chi_\alpha = M_\alpha / M_{GT}$ of different nuclear matrix elements $M_\alpha$ to the Gamow-Teller NME $M_{GT}$ calculated in [9] (pnQRPA), [6] (QRPA) and [10] (QRPA without proton-neutron pairing). Values of $\chi'_R$ [10] are calculated multiplying the $\chi_R$ values [10] by factor $m_0 c^2 R_A/4$.

| | | $^{48}$Ca | $^{76}$Ge | $^{82}$Se | $^{96}$Zr | $^{100}$Mo | $^{116}$Cd | $^{130}$Te | $^{150}$Nd |
|---|---|---|---|---|---|---|---|---|---|
| $\chi_{GT\omega}$ | [9] | – | 0.966 | 0.964 | – | 1.743 | – | 0.980 | 0.989 |
| | [6] | – | 0.951 | 0.952 | – | 0.968 | – | 0.948 | 0.943 |
| | [10] | 1.057 | 0.916 | 0.960 | 0.845 | 0.683 | 0.859 | 0.895 | – |
| $\chi_{F\omega}$ | [9] | – | –0.340 | –0.330 | – | –1.596 | – | –0.348 | –0.383 |
| | [6] | – | –0.262 | –0.258 | – | –0.304 | – | –0.268 | –0.280 |
| | [10] | –0.437 | –0.038 | –0.013 | –0.130 | –0.709 | –1.032 | 0.001 | – |
| $\chi'_{GT}$ | [9] | – | 0.645 | 0.350 | – | –1.501 | – | 0.612 | 0.584 |
| | [6] | – | 1.049 | 1.048 | – | 1.032 | – | 1.052 | 1.057 |
| | [10] | 0.975 | 1.077 | 1.050 | 1.143 | 1.174 | 1.074 | 1.097 | – |
| $\chi'_F$ | [9] | – | –0.351 | –0.339 | – | –1.522 | – | –0.345 | –0.374 |
| | [6] | – | –0.318 | –0.314 | – | –0.363 | – | –0.331 | –0.352 |
| | [10] | –0.504 | –0.035 | –0.004 | –0.168 | –0.817 | –1.173 | –0.007 | – |
| $\chi'_T$ | [9] | – | –0.203 | –0.277 | – | –1.079 | – | –0.230 | –0.270 |
| | [6] | – | –0.230 | –0.248 | – | –0.470 | – | –0.231 | –0.333 |
| | [10] | –0.212 | 0.244 | 0.079 | 0.121 | –0.477 | –0.812 | 0.282 | – |
| $\chi'_P$ | [9] | – | –0.176 | –0.176 | – | 1.549 | – | –0.155 | 0.235 |
| | [6] | – | –0.485 | –0.525 | – | 0.528 | – | –0.496 | 0.626 |
| | [10] | 0.168 | –1.147 | –0.049 | –0.836 | -3.843 | -3.891 | –1.451 | – |
| $\chi'_R$ | [9] | – | 1.192 | 1.174 | – | 5.934 | – | 1.499 | 1.647 |
| | [6] | – | 70.3 | 71.2 | – | 84.6 | – | 79.6 | 72.2 |
| | [10] | 0.486 | 0.635 | 0.419 | 0.405 | 0.379 | –0.574 | 0.586 | – |

Single electron energy distributions for the NEMO-3 $2\beta$ isotopes are shown in Fig. 1. Distributions are calculated as $\rho_1(t_1) = e_1 p_1 F(t_1, Z) e_2 p_2 F(t_2, Z) \cdot A(t_1)$ (term $B(t_1)$ disappears after integration in $\theta$) with $A(t_1)$ determined by Eq. (4) for $m$, by Eq. (13) for rhc-$\lambda$ and by Eq. (15) for rhc-$\eta$ mechanisms of $2\beta$ decay.

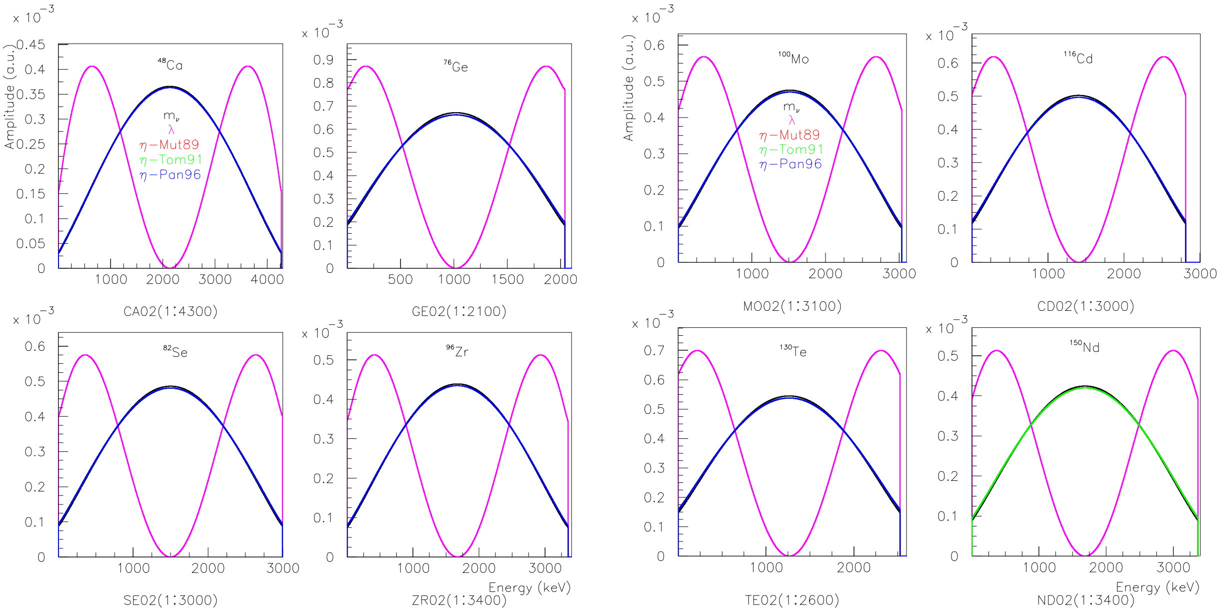

Figure 1: Single electron energy distributions for $m$, rhc-$\lambda$ and rhc-$\eta$ mechanisms of $2\beta0\nu$ decay for the NEMO-3 isotopes. Area under each curve is normalised to 1.

# 3 Discussion and conclusion

As one can see in Fig. 1, single electron energy distributions for all the NEMO-3 isotopes for the rhc-$\eta$ term are very close to the energy distributions related with the neutrino mass mechanism for NME's calculated in [9, 6, 10]. The ratio $B_\eta(t_1)/A_\eta(t_1)$, which defines angular correlation between the emitted electrons, is always positive and close to 1 (from 0.83 to 1.01) for all energies, and nuclides and NME's listed in Table 1. Thus we could suppose that for the rhc-$\eta$ term the following approximation will be good:

$$\rho_{1\theta}(t_1, \cos\theta) = e_1 p_1 F(t_1, Z) e_2 p_2 F(t_2, Z)(1 + \beta_1\beta_2 \cos\theta) \qquad (22)$$

(especially it could be very useful if for some specific isotope NME's were not calculated).

Current version of the GENBB/DECAY0 event generator gives possibility to generate $2\beta$ decay with rhc-$\eta$ term (in addition to previous $m$ and rhc-$\lambda$ mechanisms) in accordance with approximation (22), but generation with more complex expression (3) with $A_\eta$ and $B_\eta$ defined in Eqs. (15, 16) is also available. User in this case should supply values of NME's, which he likes, in external file. This file should have 3 lines; the first 2 lines are comments, and in line 3 values of NME's should be given in the following order (as in Table 1): $\chi_{GT\omega}, \chi_{F\omega}, \chi'_{GT}, \chi'_F, \chi'_T, \chi'_P, \chi'_R$[3]. Example of distribution generated by the GENBB/DECAY0 with all NME's equal to 0 except of $\chi'_T \neq 0$ is given in Fig. 2.

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[3] For cases listed in Table 1 all the files are given with the GENBB/DECAY0 code.

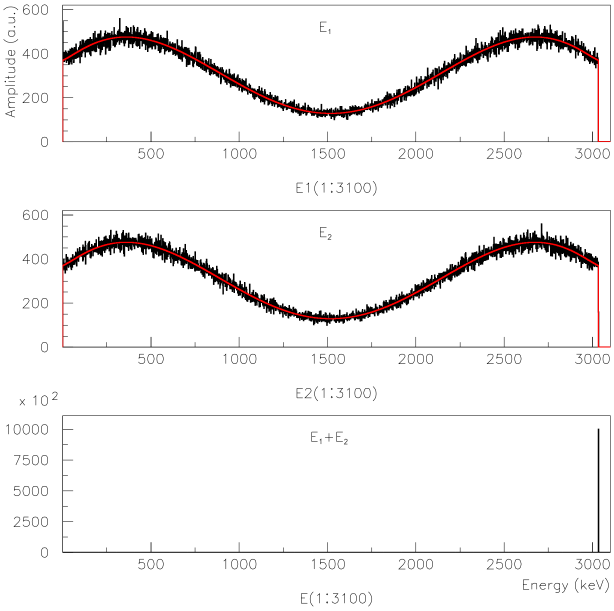

Figure 2: Single electron energy distributions and sum of electron energies – theoretical and generated with the GENBB/DECAY0 – for the rhc-$\eta$ mechanism of $^{100}$Mo $2\beta$ decay with all the NME's equal to 0 except of $\chi'_T \neq 0$.

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