NEMO note 6/93

LAL, Orsay, August 1993

Current possibilities of events generation in GENBB code

V.I.Tretyak

Abstract

Current possibilities of Monte Carlo code GENBB to generate events from background or calibration sources and events of double beta decay in NEMO installation are described. For 2β-processes it is possible to simulate decay of 8 nuclides (^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{136}Xe and ^{150}Nd) to ground and few (up to 5^{th}) excited 0^+ and 2^+ levels of daughter nuclei. 9 different modes of double beta decay are available (6 for $0^+ - 0^+$ transitions and 3 for $0^+ - 2^+$ transitions).

1 Background processes

Five nuclides were added to previous list of isotopes what can produce events in NEMO installation: ^{90}Sr, ^{90}Y, ^{22}Na, ^{88}Y, ^{207}Bi (with ^{207m}Pb). First two of them (^{90}Sr, ^{90}Y) were included only for convenience because it was possible to generate corresponding events as beta decay (^{90}Sr is pure and ^{90}Y - practically pure β-decayer). Last three isotopes (^{22}Na, ^{88}Y, ^{207}Bi+^{207m}Pb) are calibration sources. With models in list we have possibility to simulate their decay in NEMO installation to check with good statistics simulated efficiencies and spectra shapes in different channels.

So, current list of background and sources isotopes includes 18 nuclei: ^{228}Ac, ^{207}Bi+^{207m}Pb, ^{210}Bi, ^{212}Bi, ^{214}Bi, ^{60}Co, ^{137}Cs+^{137m}Ba, ^{40}K, ^{22}Na, ^{234m}Pa, ^{211}Pb, ^{212}Pb, ^{214}Pb, ^{90}Sr, ^{207}Tl, ^{208}Tl, ^{88}Y and ^{90}Y. All models are constructed in the same way [1]: the decay branches and following deexcitation process are described with 0.001% accuracy in probabilities according to nuclear schemes [2] without simplification; possibilities to emit conversion electron or e^+e^--pair instead of γ-quanta in deexcitation process are taken into account for all transitions.

GENBB code gives also the possibilities to generate the processes of α-, β-, γ- or e-emission, Compton or double Compton effect and Möller scattering from external flux of γ-quanta and electrons. Corresponding subroutines were elaborated previously by different authors (F.Laplanche and Yu.Vasilyev mainly). Subroutine for β-decay was corrected to take into account the influence of nuclear Coulomb field on emitted e^- or e^+ [1].

2 Double beta decay processes

Possibilities of 2β-events generations were extended in 3 directions:

1. 8 2β-nuclides instead of only ^{100}Mo as before: ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{136}Xe, ^{150}Nd. List of possible nuclides for the future was formed taking into account the information in [3], [4];

2. 2β-decays not only to ground state but to a few (up to the 5^{th}) excited 0^+ and 2^+ levels of daughter nucleus. Following levels of daughter nuclei are available:

^{48}Ca→^{48}Ti:	0^+_{gs} - 0 MeV;
	2^+_1 - 0.984 MeV;
	2^+_2 - 2.421 MeV;
^{76}Ge→^{76}Se:	0^+_{gs} - 0 MeV;
	2^+_1 - 0.559 MeV;
	0^+_1 - 1.122 MeV;
	2^+_2 - 1.216 MeV;
^{82}Se→^{82}Kr:	0^+_{gs} - 0 MeV;
	2^+_1 - 0.776 MeV;
	2^+_2 - 1.475 MeV;
^{96}Zr→^{96}Mo:	0^+_{gs} - 0 MeV;
	2^+_1 - 0.778 MeV;
	0^+_1 - 1.148 MeV;
	2^+_2 - 1.498 MeV;
	2^+_3 - 1.626 MeV;

$^{100}Mo \rightarrow {}^{100}Ru$: 0^+_{gs} - 0 MeV;
2^+_1 - 0.540 MeV;
0^+_1 - 1.130 MeV;
2^+_2 - 1.362 MeV;
0^+_2 - 1.741 MeV;
$^{116}Cd \rightarrow {}^{116}Sn$: 0^+_{gs} - 0 MeV;
2^+_1 - 1.294 MeV;
0^+_1 - 1.757 MeV;
0^+_2 - 2.027 MeV;
2^+_2 - 2.112 MeV;
2^+_3 - 2.225 MeV;
$^{136}Xe \rightarrow {}^{136}Ba$: 0^+_{gs} - 0 MeV;
2^+_1 - 0.819 MeV;
2^+_2 - 1.551 MeV;
0^+_1 - 1.579 MeV;
$^{150}Nd \rightarrow {}^{150}Sm$: 0^+_{gs} - 0 MeV;
2^+_1 - 0.334 MeV;
0^+_1 - 0.740 MeV;
2^+_2 - 1.046 MeV;
2^+_3 - 1.194 MeV;
0^+_2 - 1.256 MeV.

To choose on which level to put a stop, I used the big table of experimental results on probabilities of double beta processes [5]: if experimental result for given level was set whenever, a possibility to generate corresponding decay was included in list. In this way we will be able to set our own experimental limits for all these processes and possibly to improve previous results.

If 2β-decay occurs to excited level, deexcitation process with emission of γ-quanta (or conversion electrons or e^+e^--pairs) follows. Models for description of deexcitation process in daughter nuclei were constructed in the same way as in [1];

3. 9 different modes of double beta decay instead of 3 before are available now ($2n$ - two nucleon mechanism, N^* - Δ-isobar mechanism):

for $0^+ - 0^+$ transitions:

$2\beta 0\nu(m_\nu)[2n]$ - neutrinoless 2β-decay with neutrino mass;

$2\beta 0\nu(rc)[2n]$ - neutrinoless 2β-decay with right currents, $2n$-mechanism;

$2\beta 0\nu(rc)[N^*]$- neutrinoless 2β-decay with right currents, N^*-mechanism;

$2\beta 2\nu[2n]$ - 2β-decay with emission of two neutrinos;

$2\beta 0\nu M1[2n]$ - neutrinoless 2β-decay with emission of old Majoron Gelmini-Roncadelli;

$2\beta 0\nu M2[2n]$ - neutrinoless 2β-decay with emission of vector Majoron;

for $0^+ - 2^+$ transitions:

$2\beta0\nu(rc)[2n]$ - neutrinoless 2β-decay with right currents, $2n$-mechanism;

$2\beta0\nu(rc)[N^*]$ - neutrinoless 2β-decay with right currents, N^*-mechanism;

$2\beta2\nu[2n] = [N^*]$ - 2β-decay with emission of two neutrinos.

Theoretical formulae [6], [7], [8], [9] for the energy and angular distributions were used for sampling the energies and angles of electrons in 2β-decay. In cases where formulae for wanted distributions of single electron energy $\rho_1(t_1)$ or sum of electrons energies $\rho(t)$ were absent in [6], [7], [8], [9], they were obtained from basic 3-dimensional distribution $\rho_{12\theta}(t_1, t_2, cos\theta)$ in following way:

$$\rho_{12}(t_1,t_2) = \int_0^{\pi} \rho_{12\theta}(t_1,t_2,cos\theta)\, dcos\theta, \tag{1}$$

$$\rho_1(t_1) = \int_0^{t_0-t_1} \rho_{12}(t_1,t_2)\, dt_2, \tag{2}$$

$$\rho(t) = \int_0^{t} \rho_{12}(t-t_2,t_2)\, dt_2, \tag{3}$$

where t_0 - energy release in 2β-decay, t_i - kinetic energy of i^{th} electron, $t = t_1 + t_2$, θ - angle between electrons directions.

Primakoff-Rosen approximation was used to obtain all formulae in analytical way (on adequacy of Primakoff-Rosen approximation see previous NEMO papers [10], [11], [12]). All results were checked with the help of MATHEMATICA package for analytical calculations [13].

Used formulae for energy and angular distributions of electrons in different modes of 2β-decay and the results of 2β-events generation by GENBB code are shown below.

2.1 $2\beta0\nu$ with neutrino mass, $0^+ - 0^+$ transition, $2n$-mechanism

$$\rho_{12\theta}(t_1,t_2,cos\theta) = (t_1+1)^2(t_2+1)^2\delta(t_0-t_1-t_2)(1-\beta_1\beta_2 cos\theta), \tag{4}$$

$$\rho_{12}(t_1,t_2) = (t_1+1)^2(t_2+1)^2\delta(t_0-t_1-t_2), \tag{5}$$

$$\rho_1(t_1) = (t_1+1)^2(t_0+1-t_1)^2, \tag{6}$$

$$\rho(t) = \delta(t_0-t), \tag{7}$$

where t_i - kinetic energy of i^{th} electron in units of electron mass; t_0 - available for electrons energy ($Q_{\beta\beta}$ for decay to ground state and $Q_{\beta\beta} - E_{level}$ for decay to excited state of daughter nucleus with energy E_{level}) in the same units; $t = t_1 + t_2$; β_i - velocities of electrons, $\beta_i = \sqrt{t_i(t_i+2)}/(t_i+1)$; θ - angle between electrons directions.

Results of generation of 50000 events and their fit by corresponding theoretical energy distributions are shown in fig.1. Angular distribution see in fig.10.

2.2 $2\beta0\nu$ with right currents, $0^+ - 0^+$ transition, $2n$-mechanism

Exact expression for $\rho_{12\theta}(t_1,t_2,cos\theta)$ [6], [7] is quite big and depends on different nuclear matrix elements. So, not only Primakoff-Rosen approximation but also further recommended in [6], [7] simplifications were used in this case to be independent from not very well known matrix elements and to use the same formulae for all 2β-nuclides.

$$\rho_{12\theta}(t_1,t_2,cos\theta) = (t_1+1)^2(t_2+1)^2(t_1-t_2)^2\delta(t_0-t_1-t_2)(1+\beta_1\beta_2 cos\theta), \tag{8}$$

$$\rho_{12}(t_1,t_2) = (t_1+1)^2(t_2+1)^2(t_1-t_2)^2\delta(t_0-t_1-t_2), \tag{9}$$

$$\rho_1(t_1) = (t_1+1)^2(t_0+1-t_1)^2(t_0-2t_1)^2, \quad (10)$$

$$\rho(t) = \delta(t_0 - t). \quad (11)$$

Results of generation of 50000 events and their fit by corresponding theoretical energy distributions are shown in fig.2. Angular distribution see in fig.10.

2.3 $2\beta0\nu$ with right currents, $0^+ - 0^+$ transition, N^*-mechanism

$$\rho_{12\theta}(t_1, t_2, cos\theta) = \varepsilon_1\varepsilon_2[2p_1^2p_2^2cos^2\theta - p_1p_2cos\theta[(\varepsilon_1+\varepsilon_2)^2 + 4(\varepsilon_1\varepsilon_2+1)]+ \quad (12)$$
$$+3(\varepsilon_1\varepsilon_2+1)(p_1^2+p_2^2)]\delta(t_0-t_1-t_2),$$

$$\rho_{12}(t_1, t_2) = \varepsilon_1\varepsilon_2[4p_1^2p_2^2 + 9(\varepsilon_1\varepsilon_2+1)(p_1^2+p_2^2)]\delta(t_0-t_1-t_2), \quad (13)$$

$$\rho_1(t_1) = \varepsilon_1\varepsilon_2[4p_1^2p_2^2 + 9(\varepsilon_1\varepsilon_2+1)(p_1^2+p_2^2)], \quad (14)$$

$$\rho(t) = \delta(t_0 - t), \quad (15)$$

where $\varepsilon_i = t_i + 1$, $p_i^2 = t_i(t_i+2)$, $t_2 = t_0 - t_1$.

Results of generation of 50000 events and their fit by corresponding theoretical energy distributions are shown in fig.3. Angular distribution see in fig.10.

2.4 $2\beta2\nu$, $0^+ - 0^+$ transition, $2n$-mechanism

$$\rho_{12\theta}(t_1, t_2, cos\theta) = (t_1+1)^2(t_2+1)^2(t_0-t_1-t_2)^5(1-\beta_1\beta_2cos\theta), \quad (16)$$

$$\rho_{12}(t_1, t_2) = (t_1+1)^2(t_2+1)^2(t_0-t_1-t_2)^5, \quad (17)$$

$$\rho_1(t_1) = (t_1+1)^2(t_0-t_1)^6[(t_0-t_1)^2 + 8(t_0-t_1) + 28], \quad (18)$$

$$\rho(t) = (t^4 + 10t^3 + 40t^2 + 60t + 30)t(t_0-t)^5. \quad (19)$$

Results of generation of 50000 events and their fit by corresponding theoretical energy distributions are shown in fig.4. Angular distribution see in fig.10.

2.5 $2\beta0\nu$ with emission of old Majoron, $0^+ - 0^+$ transition, $2n$-mechanism

Old Majoron means Gelmini-Roncadelli Majoron [14].

$$\rho_{12\theta}(t_1, t_2, cos\theta) = (t_1+1)^2(t_2+1)^2(t_0-t_1-t_2)(1-\beta_1\beta_2cos\theta), \quad (20)$$

$$\rho_{12}(t_1, t_2) = (t_1+1)^2(t_2+1)^2(t_0-t_1-t_2), \quad (21)$$

$$\rho_1(t_1) = (t_1+1)^2[(t_0+1-t_1)^4 - 4(t_0+1-t_1) + 3], \quad (22)$$

$$\rho(t) = (t^4 + 10t^3 + 40t^2 + 60t + 30)t(t_0-t). \quad (23)$$

Results of generation of 50000 events and their fit by corresponding theoretical energy distributions are shown in fig.5. Angular distribution see in fig.10.

2.6 $2\beta 0\nu$ with emission of vector Majoron, $0^+ - 0^+$ transition, $2n$-mechanism

Information of [8], [9] about $\rho_{12\theta}(t_1, t_2, cos\theta)$ was used.

$$\rho_{12\theta}(t_1, t_2, cos\theta) = (t_1+1)^2(t_2+1)^2[(t_0-t_1-t_2)^2 - m^2]^{3/2}(1-\beta_1\beta_2 cos\theta), \quad (24)$$

where m - mass of vector Majoron. If $m \neq 0$, corresponding analytical expressions for $\rho_1(t_1)$ and $\rho(t)$ have length in few lines, so currently I decided to use $m = 0$ only. In this case

$$\rho_{12\theta}(t_1, t_2, cos\theta) = (t_1+1)^2(t_2+1)^2(t_0-t_1-t_2)^3(1-\beta_1\beta_2 cos\theta), \quad (25)$$

$$\rho_{12}(t_1, t_2) = (t_1+1)^2(t_2+1)^2(t_0-t_1-t_2)^3, \quad (26)$$

$$\rho_1(t_1) = (t_1+1)^2(t_0-t_1)^4[(t_0-t_1)^2 + 6(t_0-t_1) + 15]; \quad (27)$$

$$\rho(t) = (t^4 + 10t^3 + 40t^2 + 60t + 30)t(t_0-t)^3. \quad (28)$$

Results of generation of 50000 events and their fit by corresponding theoretical energy distributions are shown in fig.6. Angular distribution see in fig.10.

2.7 $2\beta 0\nu$ with right currents, $0^+ - 2^+$ transition, $2n$-mechanism

$$\rho_{12\theta}(t_1, t_2, cos\theta) = \varepsilon_1\varepsilon_2[3p_1^2p_2^2cos^2\theta - p_1p_2cos\theta[10(\varepsilon_1\varepsilon_2+1) + p_1^2 + p_2^2] + \quad (29)$$
$$+5(\varepsilon_1\varepsilon_2+1)(p_1^2+p_2^2) - p_1^2p_2^2]\delta(t_0-t_1-t_2),$$

$$\rho_{12}(t_1, t_2) = \varepsilon_1\varepsilon_2[p_1^2p_2^2 + 5(\varepsilon_1\varepsilon_2+1)(p_1^2+p_2^2)]\delta(t_0-t_1-t_2), \quad (30)$$

$$\rho_1(t_1) = \varepsilon_1\varepsilon_2[p_1^2p_2^2 + 5(\varepsilon_1\varepsilon_2+1)(p_1^2+p_2^2)], \quad (31)$$

$$\rho(t) = \delta(t_0-t), \quad (32)$$

where $t_2 = t_0 - t_1$.

Results of generation of 50000 events and their fit by corresponding theoretical energy distributions are shown in fig.7. Angular distribution see in fig.10.

2.8 $2\beta 0\nu$ with right currents, $0^+ - 2^+$ transition, N^*-mechanism

In this case theoretical formulae are the same as for $0^+ - 0^+$ transition (see section 2.3). However t_0 value is not the same and one or more γ-quanta are emitted.

Results of generation of 50000 events and their fit by corresponding theoretical energy distributions are shown in fig.8. Angular distribution see in fig.10.

2.9 $2\beta 2\nu$, $0^+ - 2^+$ transition, $2n$-mechanism

Up to now given mode of 2β-decay was considered as suppressed but in recent paper [15] this result is revised.

$$\rho_{12\theta}(t_1, t_2, cos\theta) = (t_1+1)^2(t_2+1)^2(t_1-t_2)^2(t_0-t_1-t_2)^7(1 + \frac{1}{3}\beta_1\beta_2 cos\theta), \quad (33)$$

$$\rho_{12}(t_1, t_2) = (t_1+1)^2(t_2+1)^2(t_1-t_2)^2(t_0-t_1-t_2)^7. \quad (34)$$

For $\rho_1(t_1)$ two slightly different expressions were obtained in [6] and [7] accordingly:

$$\rho_1(t_1) = (t_1+1)^2(t_0-t_1)^8[(t_0-t_1)^4 - 6t_1(t_0-t_1)^3 + 11(t_1^2-4t_1+1)(t_0-t_1)^2 + \tag{35}$$

$$+110t_1(t_1-1)(t_0-t_1) + 495t_1^2 + 6(t_0-t_1)^3],$$

$$\rho_1(t_1) = (t_1+1)^2(t_0-t_1)^8[(t_0-t_1)^4 - 6t_1(t_0-t_1)^3 + 11(t_1^2-4t_1+1)(t_0-t_1)^2 + \tag{36}$$

$$+110t_1(t_1-1)(t_0-t_1) + 459t_1^2].$$

Check-up of these results revealed that formula (36) is not correct and formula (35) was used for 2β-events generation (in this case MATHEMATICA package [13] was especially useful).

$$\rho(t) = t^3(t^4 + 14t^3 + 84t^2 + 140t + 70)(t_0 - t)^7. \tag{37}$$

The formulae for N^*-mechanism in this mode of decay are the same as for $2n$-mechanism.

Results of generation of 50000 events and their fit by corresponding theoretical energy distributions are shown in fig.9. Angular distribution see in fig.10.

2.10 Transitions to excited levels

Different modes of double beta decay can occurs not only to ground 0^+ or first 2^+ state of daughter nucleus but to higher states also (so, we can produce a lot of restrictions ...). In this case t_0 value in formulae becomes smaller and angular and energy distributions become different as compared with fig.1-10. One or more γ-quanta are emitted also. Energy spectra of emitted γ-quanta together with spectra of sum of electrons energies for $2\beta2\nu$-decays are shown in fig.11 for ^{100}Mo and in fig.12 - for the next possible isotope ^{116}Cd to be measured in NEMO installation for all considered currently levels in GENBB code (10000 events for each of them).

References

[1] V.I.Tretyak, "Models of decay of natural radioactive nuclides", NEMO note 2/92, Orsay, 1992, 43 p.

[2] "Table of isotopes", ed. by C.M.Lederer and V.S.Shirley, 7^{th} ed., 1978

[3] Proceedings of NEMO Collaboration meetings, 1992-1993

[4] A.S.Barabash, private communication

[5] Yu.G.Zdesenko, V.I.Tretyak, "Experimental limits on the probabilities of double beta processes", preprint KINR-92-15, Kiev, 1992, 24 p.

[6] M.Doi, T.Kotani, H.Nishiura et al., "Neutrino mass, the right-handed interaction and the double beta decay", Progr.Theor.Phys. 66(1981)1739-1788

[7] M.G.Shchepkin, "Double beta decay and the neutrino mass", Sov.Phys.Usp. 27(1984)555-578

[8] C.D.Carone, "Double beta decay with vector majorons", Phys.Lett.B 308(1993)85-88

[9] C.D.Carone, private communication

[10] D.Dassie, P.Mennrath, "Spectre theorique $\beta\beta(2\nu)$ du ^{100}Mo", preprint CENBG 9302, Bordeaux, 1993, 6 p.

[11] J.E.Campagne, "New Monte-Carlo based on Doi et al. formulae for $\beta\beta2\nu$, $\beta\beta0\nu$ and $\beta\beta\chi$ processes", NEMO note 2/93, Orsay, 1993, 23 p.

[12] V.I.Tretyak, "More sophisticated initial energy distribution for simulation of $2\beta2\nu$-decay", Proc. NEMO Collaboration meeting, Kiev, 19-23 April, 1993, p.81-85

[13] S.Wolfram, "Mathematica: a system for doing mathematics by computer", 2^{nd} ed., 1991, 961 p.

[14] G.B.Gelmini, M.Roncadelli, "Left-handed neutrino mass scale and spontaneously broken lepton number", Phys.Lett.B 99(1981)411-415

[15] J.Suhonen, O.Civitarese, "Two-neutrino $\beta\beta$ decay to excited states. The $0^+ - 2^+$ decay of ^{136}Xe", Phys.Lett.B 308(1993)212-215

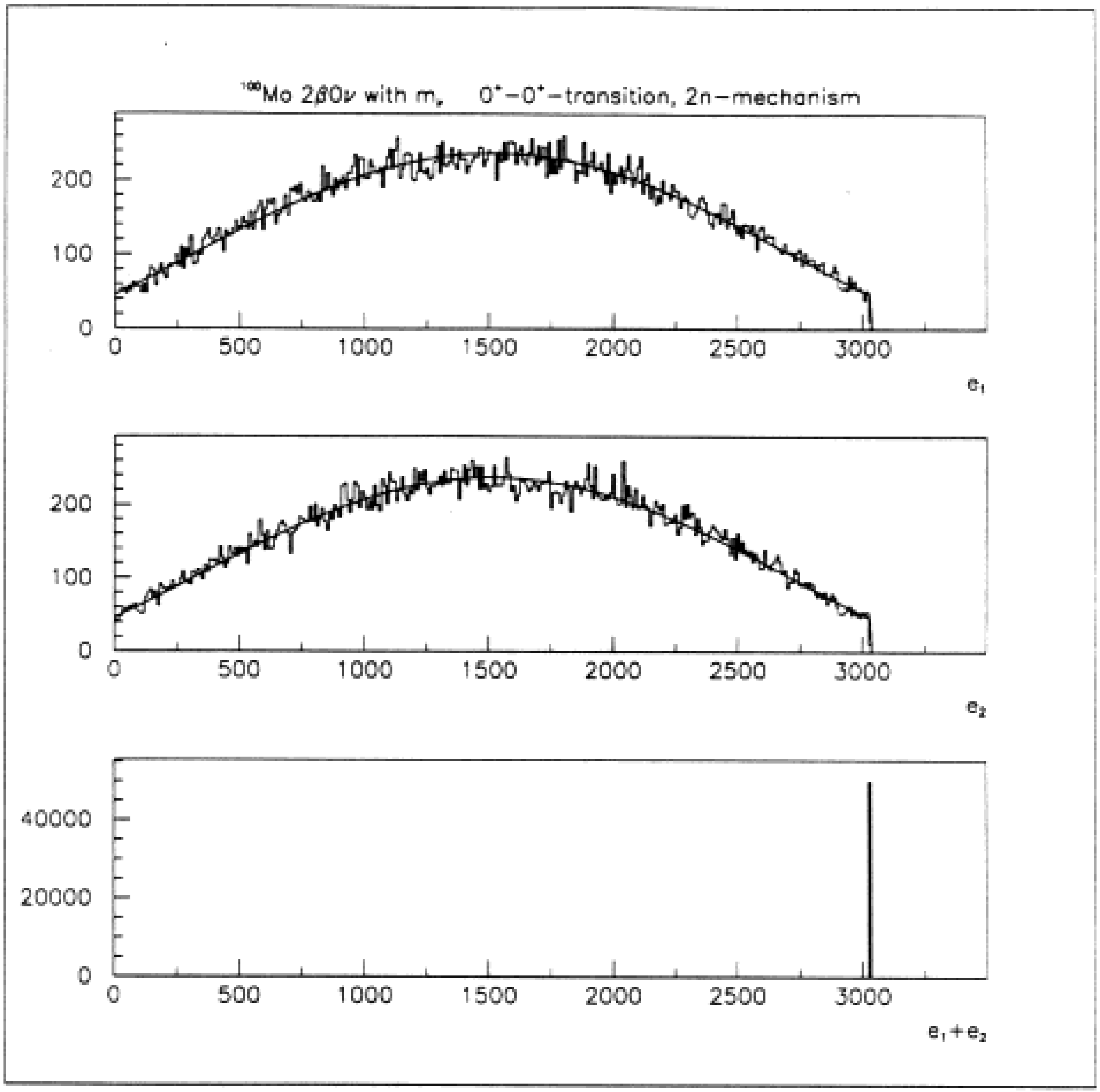

Figure 1: Energy distributions of electrons in $2\beta 0\nu$-decay with neutrino mass ($0^+ - 0^+$-transition, $2n$-mechanism). Spectra of single electrons (e_1 and e_2), spectrum of sum of energies ($e_1 + e_2$) for 50000 generated events and their fit by theoretical distributions are shown.

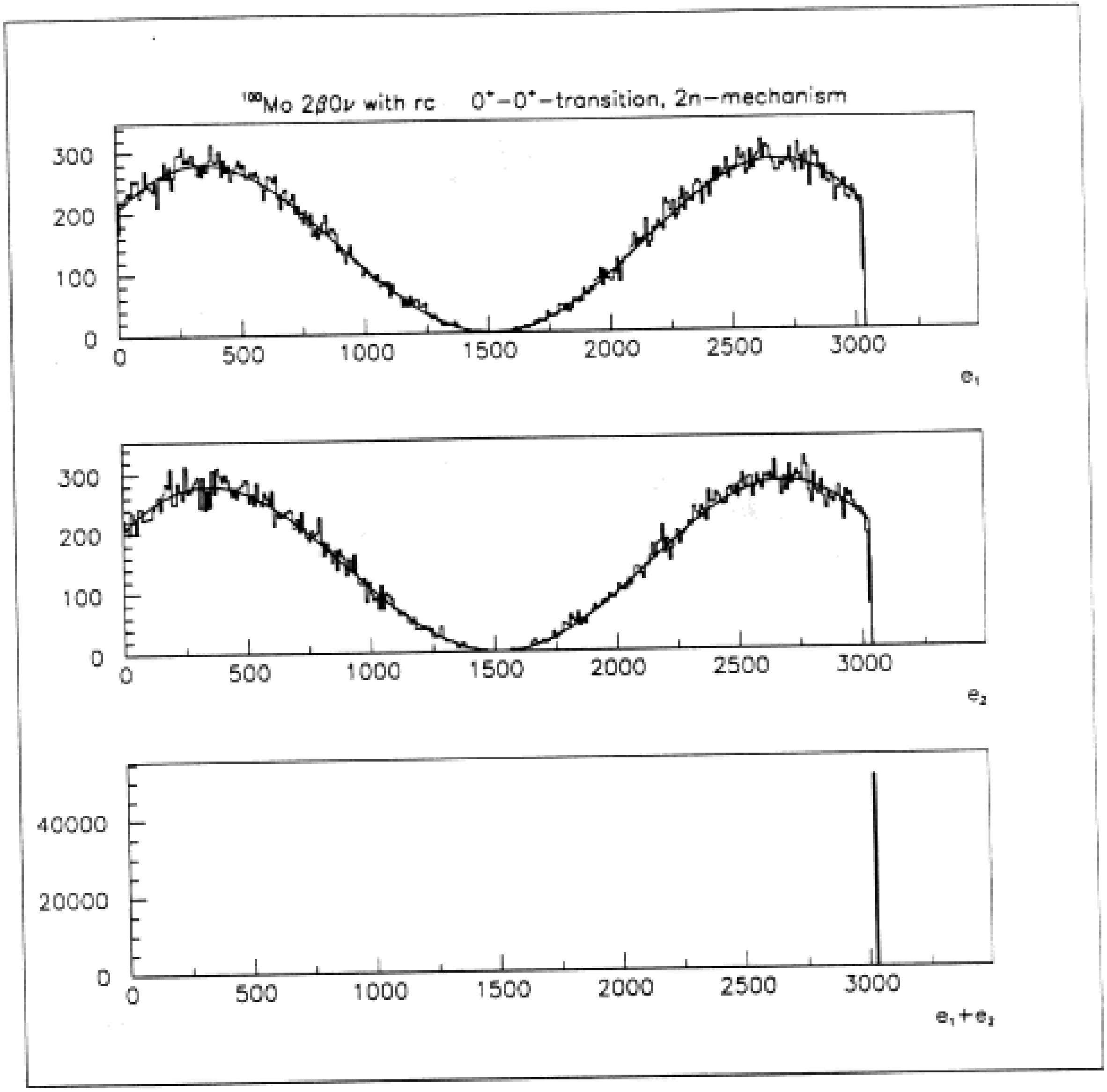

Figure 2: Energy distributions of electrons in $2\beta0\nu$-decay with right currents (0^+-0^+-transition, $2n$-mechanism). Spectra of single electrons (e_1 and e_2), spectrum of sum of energies (e_1+e_2) for 50000 generated events and their fit by theoretical distributions are shown.

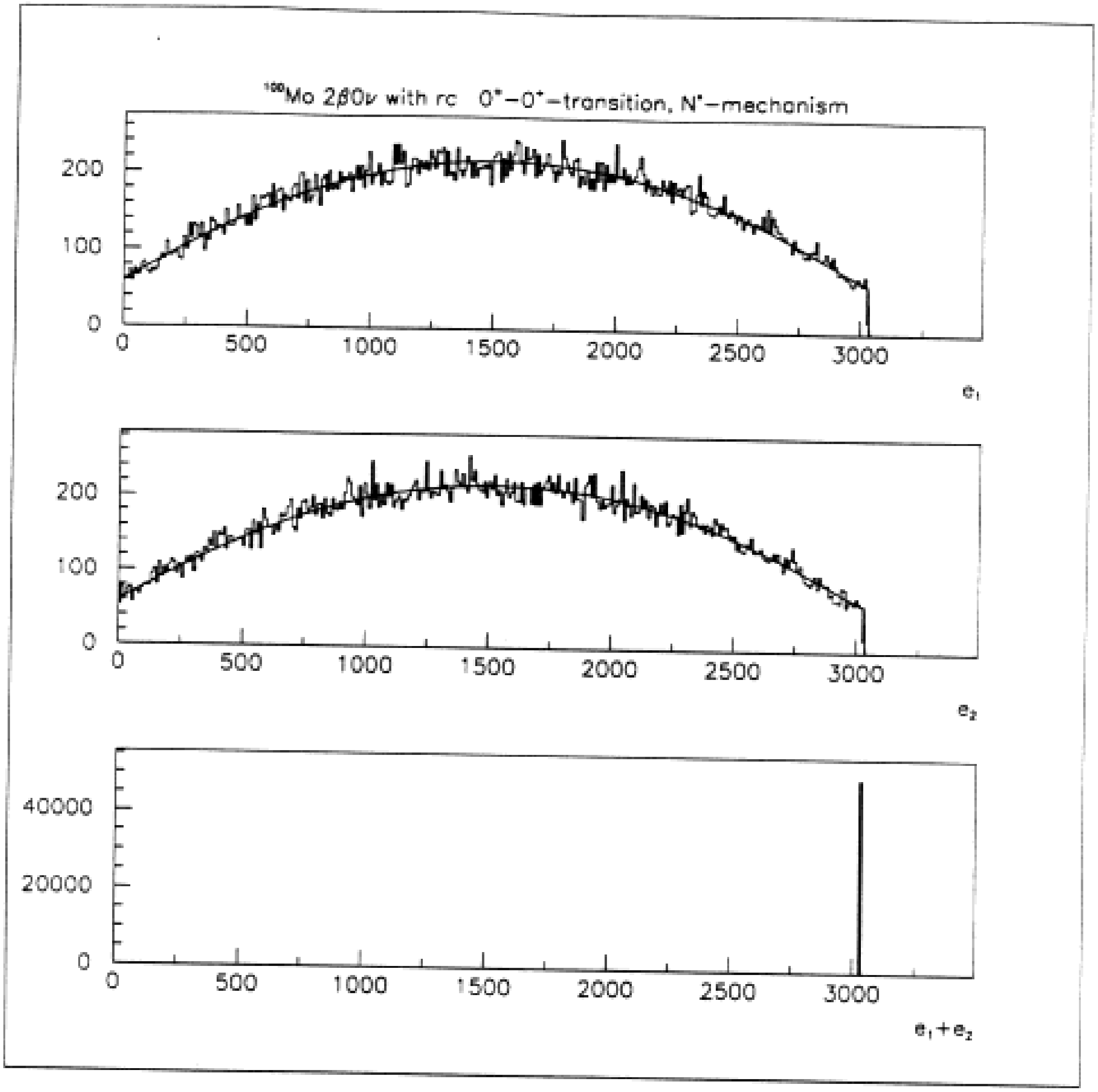

Figure 3: Energy distributions of electrons in $2\beta0\nu$-decay with right currents (0^+-0^+-transition, N^*-mechanism). Spectra of single electrons (e_1 and e_2), spectrum of sum of energies (e_1+e_2) for 50000 generated events and their fit by theoretical distributions are shown.

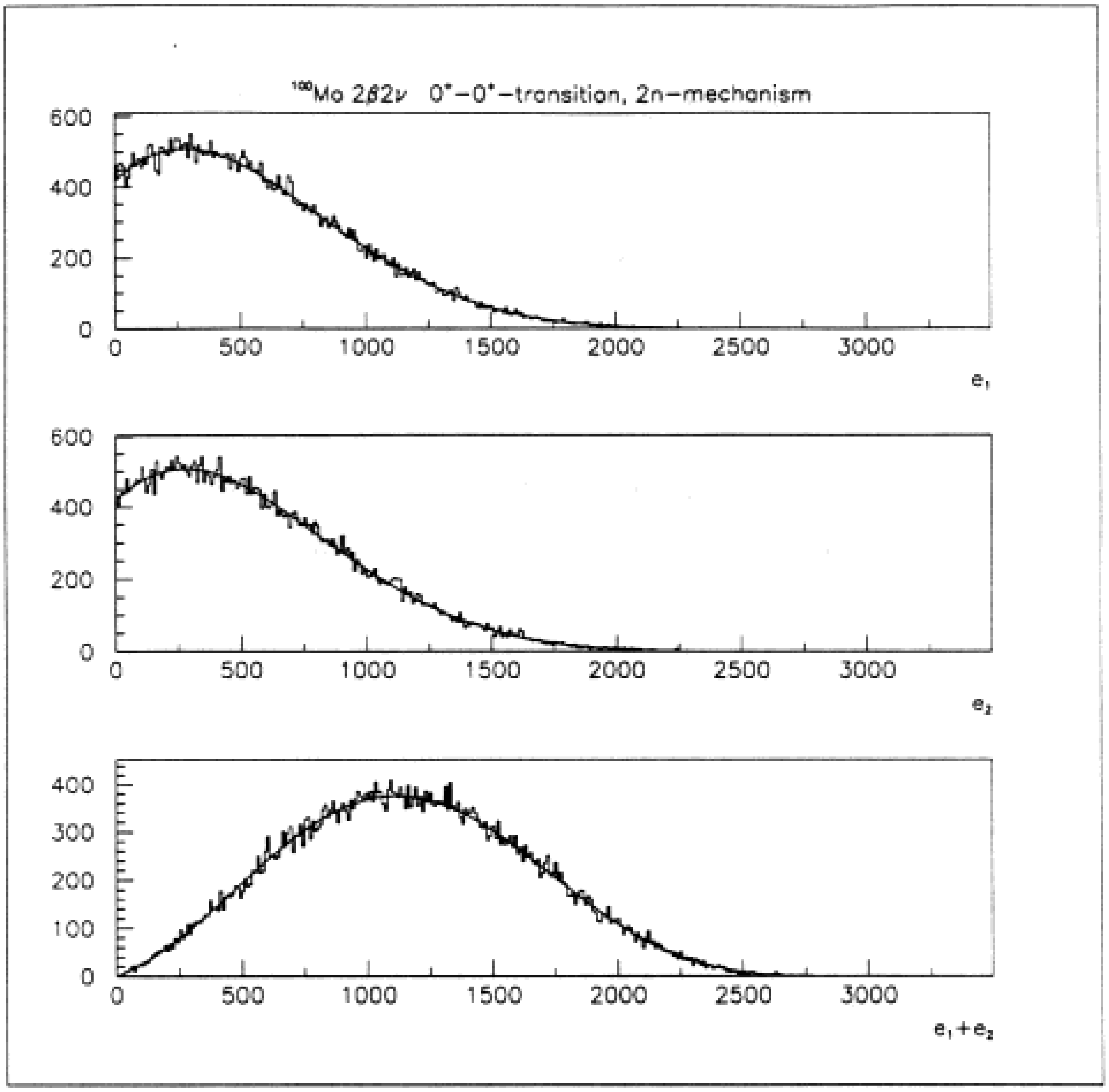

Figure 4: Energy distributions of electrons in $2\beta2\nu$-decay ($0^+ - 0^+$-transition, $2n$-mechanism). Spectra of single electrons (e_1 and e_2), spectrum of sum of energies ($e_1 + e_2$) for 50000 generated events and their fit by theoretical distributions are shown.

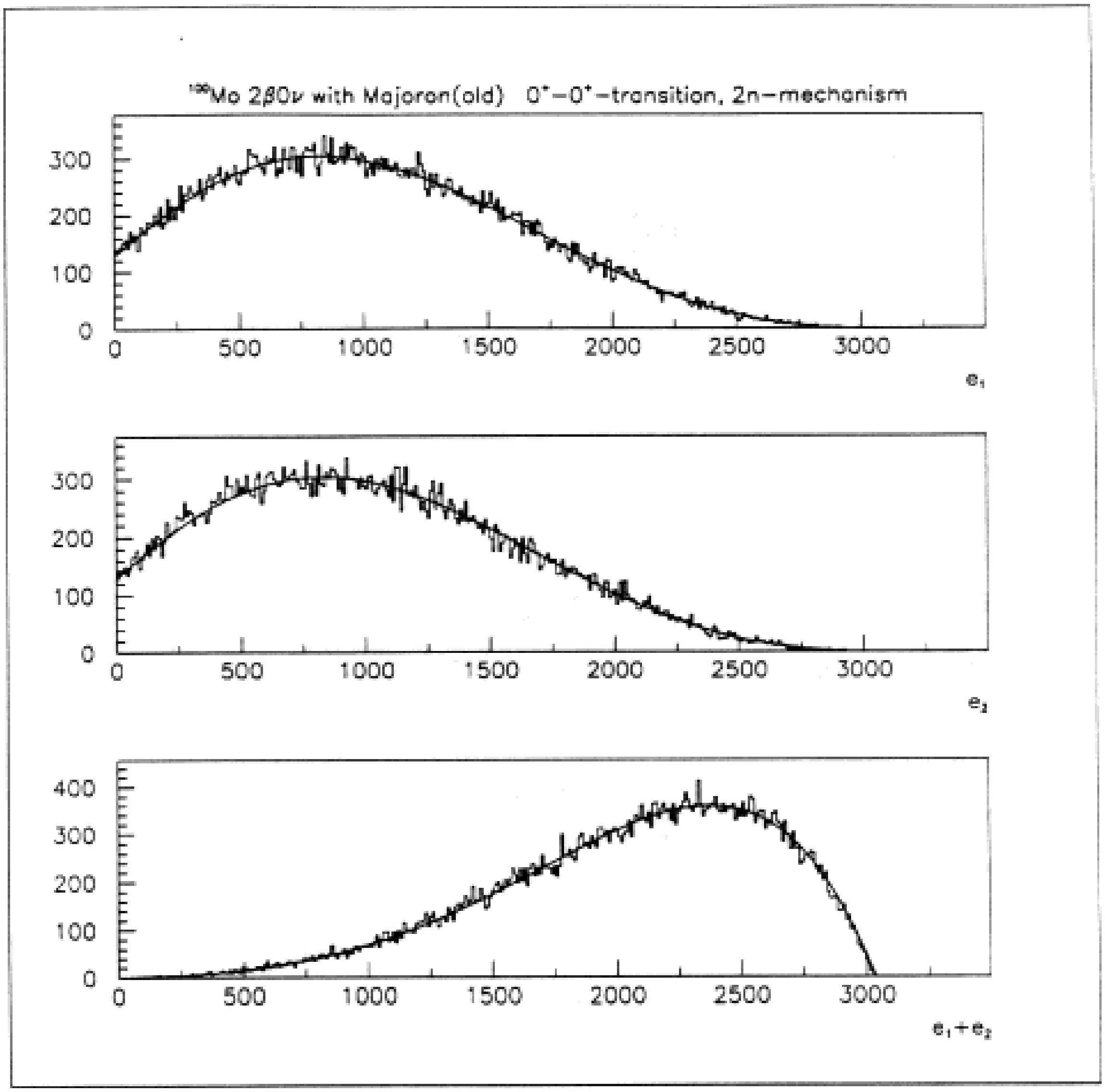

Figure 5: Energy distributions of electrons in $2\beta0\nu$-decay with old Majoron Gelmini-Roncadelli ($0^+ - 0^+$-transition, $2n$-mechanism). Spectra of single electrons (e_1 and e_2), spectrum of sum of energies ($e_1 + e_2$) for 50000 generated events and their fit by theoretical distributions are shown.

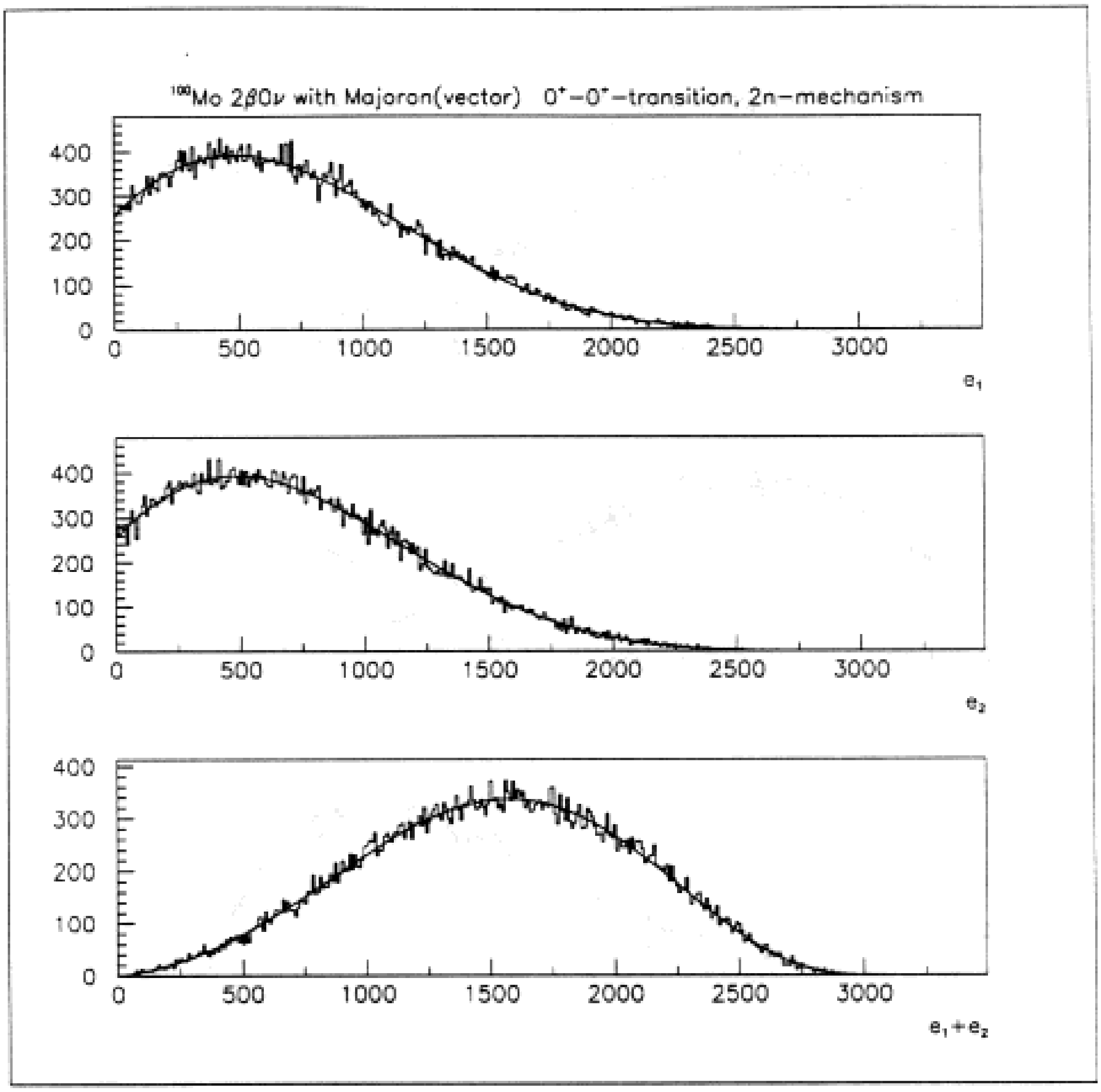

Figure 6: Energy distributions of electrons in $2\beta0\nu$-decay with vector Majoron ($0^+ - 0^+$-transition, $2n$-mechanism). Spectra of single electrons (e_1 and e_2), spectrum of sum of energies ($e_1 + e_2$) for 50000 generated events and their fit by theoretical distributions are shown.

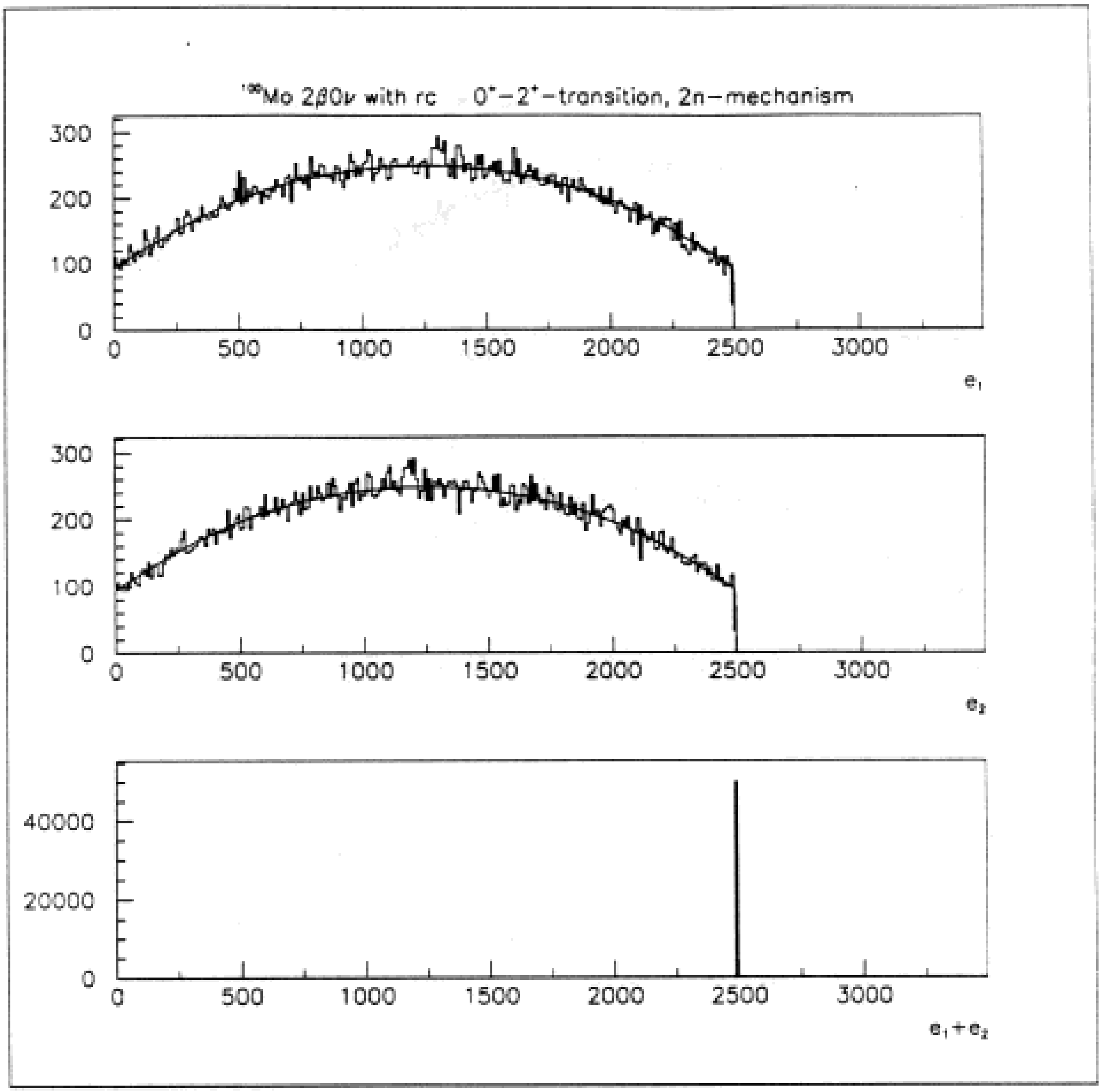

Figure 7: Energy distributions of electrons in $2\beta0\nu$-decay with right currents ($0^+ - 2^+$-transition, $2n$-mechanism). Spectra of single electrons (e_1 and e_2), spectrum of sum of energies ($e_1 + e_2$) for 50000 generated events and their fit by theoretical distributions are shown.

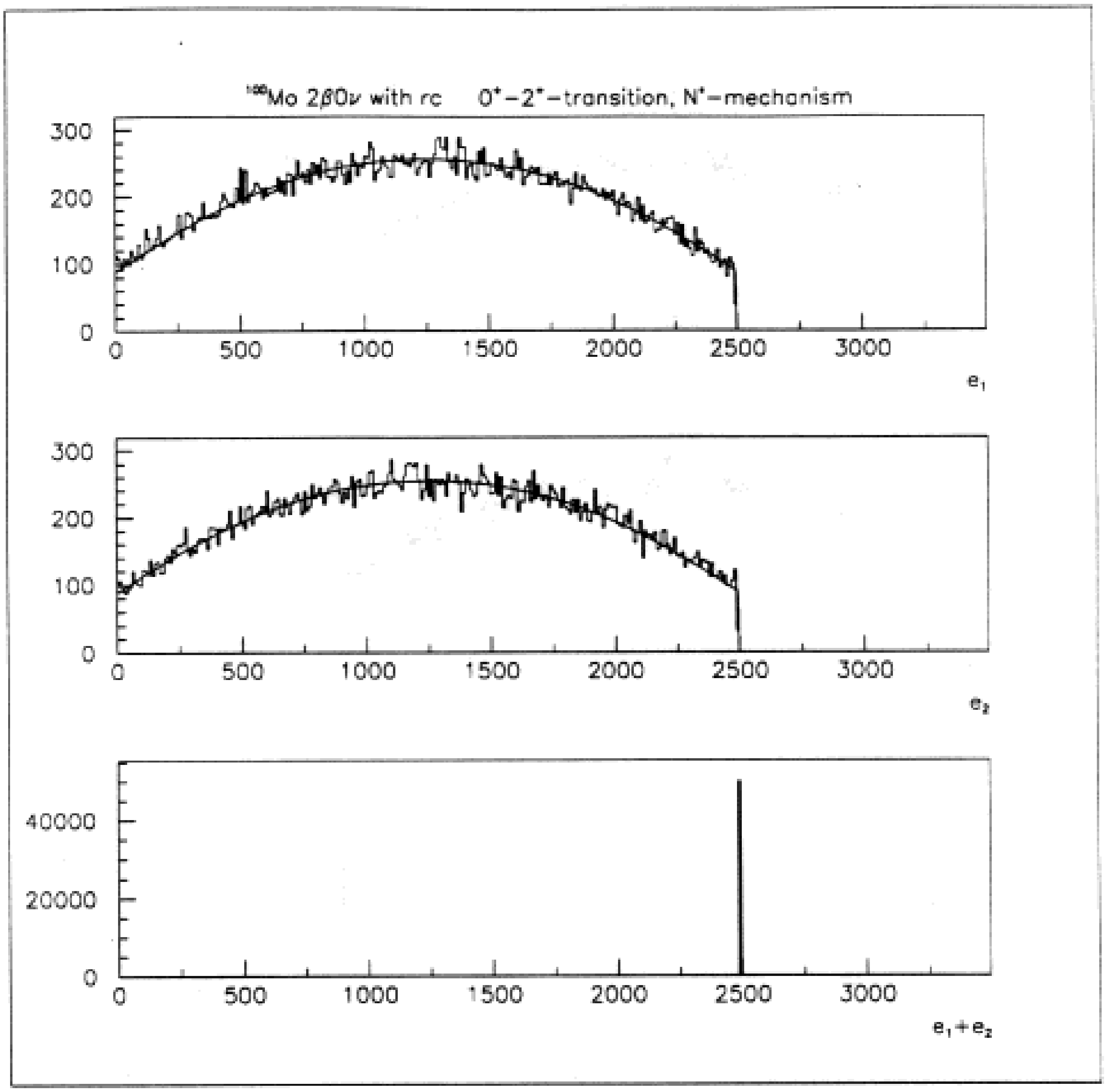

Figure 8: Energy distributions of electrons in $2\beta 0\nu$-decay with right currents ($0^+ - 2^+$-transition, N^*-mechanism). Spectra of single electrons (e_1 and e_2), spectrum of sum of energies ($e_1 + e_2$) for 50000 generated events and their fit by theoretical distributions are shown.

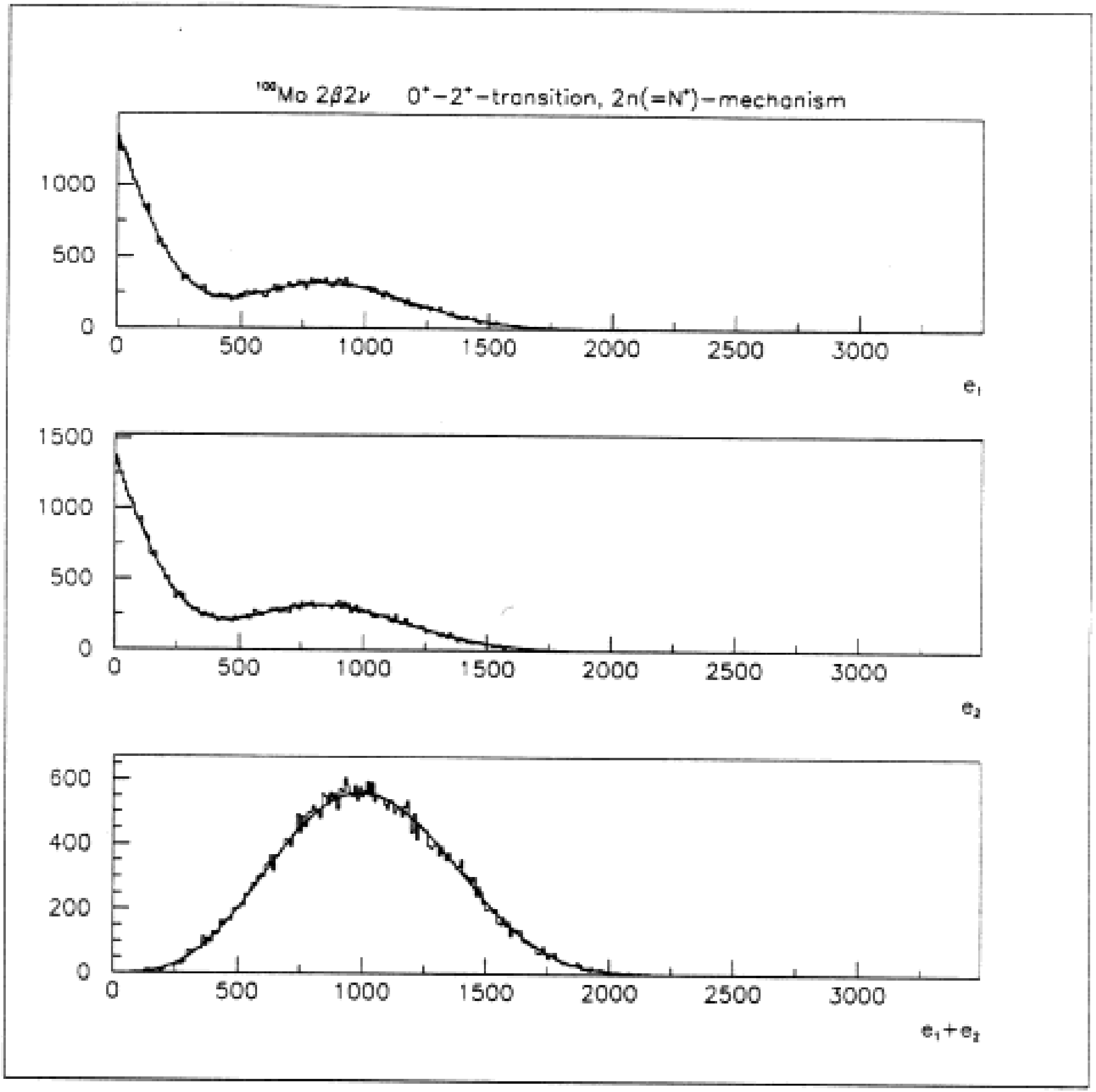

Figure 9: Energy distributions of electrons in $2\beta2\nu$-decay (0^+ – 2^+-transition, $2n$- or N^*-mechanism). Spectra of single electrons (e_1 and e_2), spectrum of sum of energies ($e_1 + e_2$) for 50000 generated events and their fit by theoretical distributions are shown.

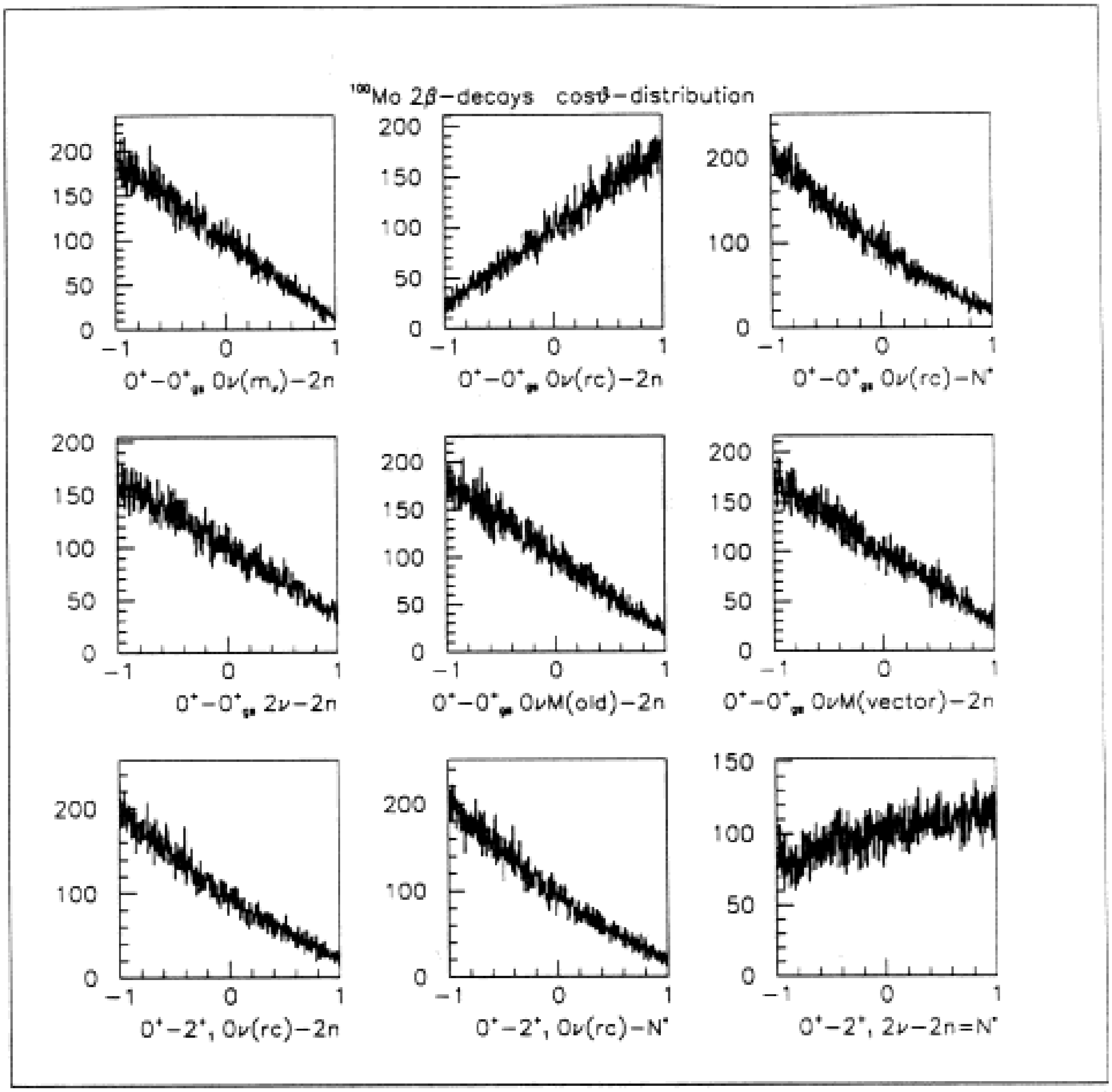

Figure 10: Distributions of angle between two electrons in different modes of ^{100}Mo 2β-decay.

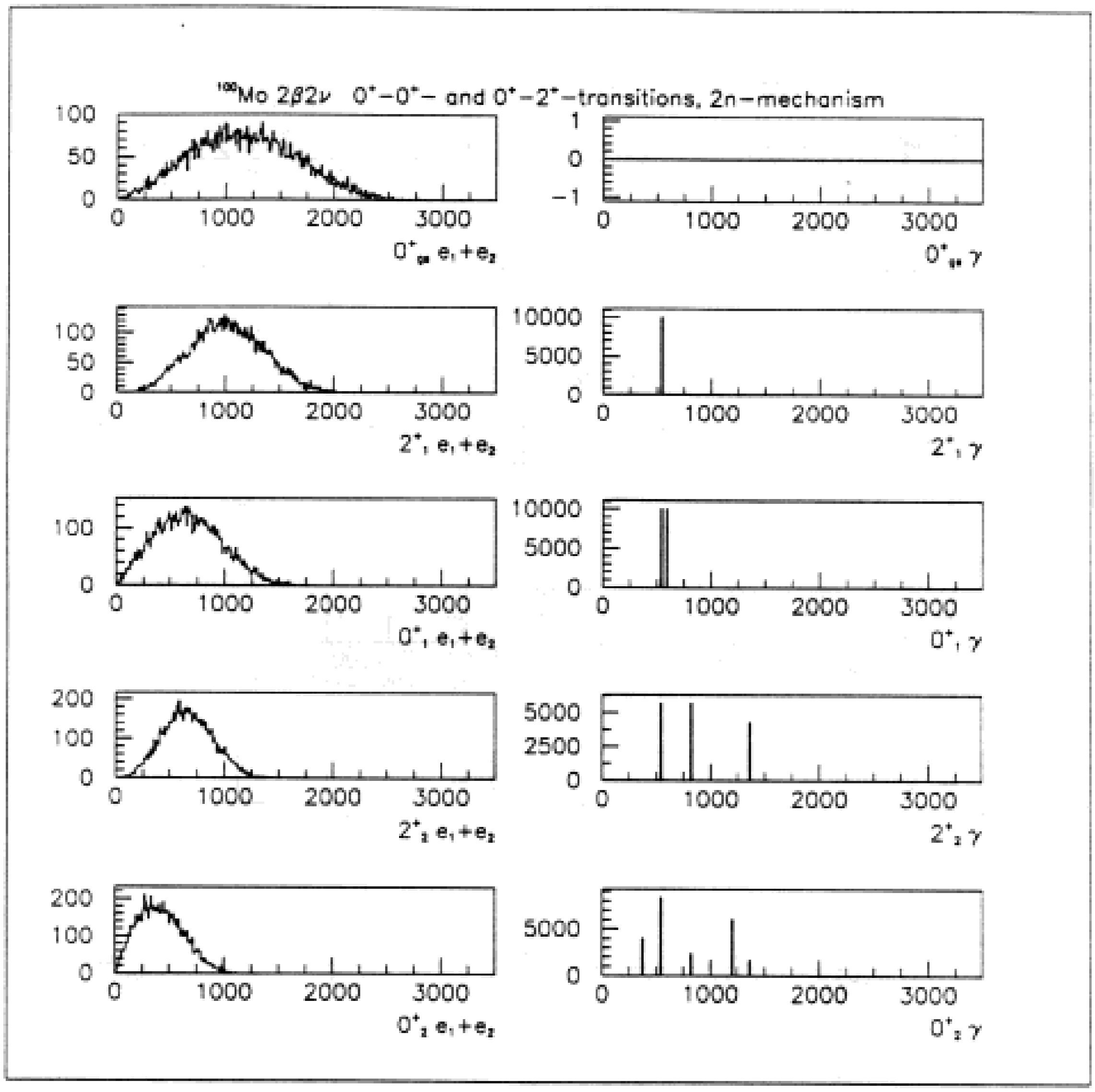

Figure 11: Energy spectra of electrons and γ-quanta in $2\beta 2\nu$-decay of ^{100}Mo to ground 0^+_{gs} and excited 2^+_1, 0^+_1, 2^+_2, 0^+_2 states of ^{100}Ru.

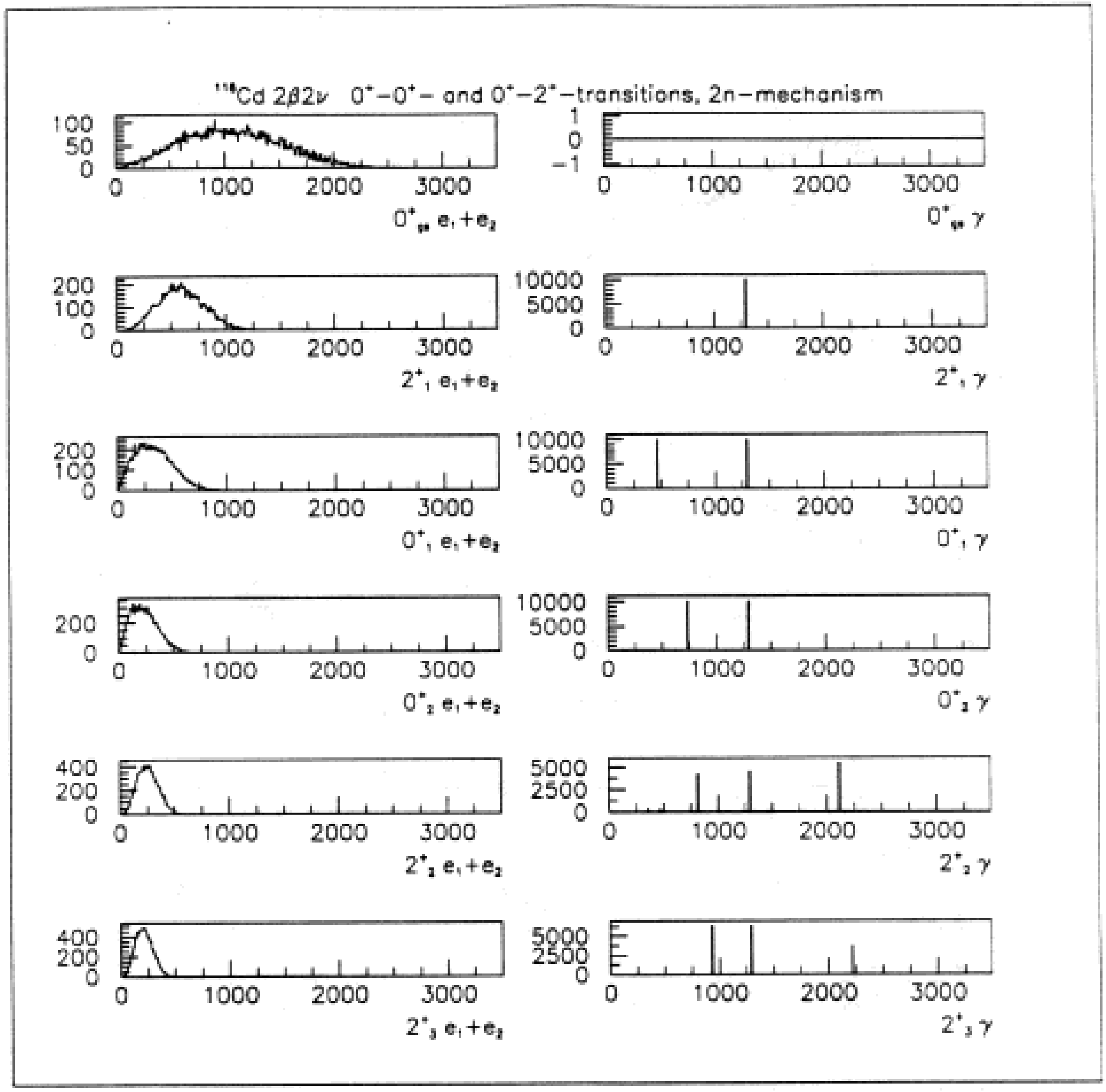

Figure 12: Energy spectra of electrons and γ-quanta in $2\beta2\nu$-decay of ^{116}Cd to ground 0^+_{gs} and excited 2^+_1, 0^+_1, 0^+_2, 2^+_2, 2^+_3 states of ^{116}Sn.