Group NEMO
Note 2/92

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# MODELS OF DECAY OF NATURAL RADIOACTIVE NUCLIDES

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CONTENTS

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## 1. Analysis of danger to imitate double beta decay of 100-Mo by internal radioactive impurities of Mo-foil

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The presence of radioactive impurities in Mo foil allows maybe not to imitate completely the double beta decay of 100-Mo but at least to give double electron events in suitable regions of energy of electrons and angle between the tracks. Beta decaying nuclides are more dangerous than alpha decaying because they give one electron at once. Second electron can be created by means of a quite big number of processes:

(1) after the Moller scattering of first electron on the atomic shell electrons inside the foil (or after the Bhabba scattering in the case of positron as incident particle);

(2) the daughter nucleus is created usually in excited state after the decay. Gamma quanta are emitted in deexcitation process mainly but conversion electrons or electron-positron pair (if the energy release is greater 1.022 MeV) can be emitted also instead of gammas;

(3) deexcitation gamma quantum during its travel in foil can produce second electron in the result of Compton effect or photoeffect; if gamma energy is greater 1.022 MeV the pair e+e- can be created.

Alpha emitters can't be completely excluded because of above-mentioned processes (2) and (3) and of process (1) afterwards (with addition of double Compton effect, for example).

As we know (see for instance [1]) Mo samples (both with natural composition and enriched in 100-Mo) have impurities in nuclides of natural radioactive families 232-Th, 235-U and 238-U and also 40-K, 60-Co and 137-Cs. So we need:

(1) to estimate the potential degree of danger to imitate double beta decay for each nuclide from 232-Th, 235-U and 238-U chains and 40-K, 60-Co, 137-Cs;

(5) to simulate the decay of dangerous isotopes in order to know the efficiency and response function of NEMO detector for corresponding double electron events.

The natural radioactive chains of 232-Th, 235-U and 238-U are shown in well-known fig. 1 and characteristics of different nuclides are listed in table 1 in following order:

- parent nuclide;
- probability for its appearance in the decay chain;
- modes of decay (a/b/g - alpha/beta/gamma) and their probabilities;
- daughter nuclide;
- energy release (MeV);
- maximum energy of excited level of daughter nucleus occupied in the decay of parent nucleus;
- degree of danger (in my opinion, of course) from weak (*) to very strong (****); last can give events in the region of neutrinoless mode of 100-Mo double beta decay.

One can see from table 1 that we have:

- 2 nuclides with ****-degree of danger (energy release more than 3 MeV and high probability for this mode of decay)
  - 208-Tl,
  - 214-Bi;
- 4 nuclides with ***-degree of danger (Q is near 2 MeV, high probability)
  - 228-Ac,
  - 212-Bi,
  - 234m-Pa,
  - 60-Co;
- 6 nuclides with **-degree of danger (Q is near 1 MeV, high probability)
  - 211-Pb,
  - 207-Tl,
  - 214-Pb,
  - 210-Bi,
  - 40-K,
  - 137-Cs.

It is interesting to note that in 238-U chain we have the beta decaying nuclide with energy release even greater than that of 208-Tl - 210-Tl (5.487

and 4.992 MeV correspondingly) but the probability of 210-Tl appearance in the decay chain is quite low (0.021%).

I think the decays of all nuclides from **** to ** must be simulated in NEMO detector - all of them can give events with two electron tracks at least in the region of double neutrino mode of 100-Mo double beta decay.

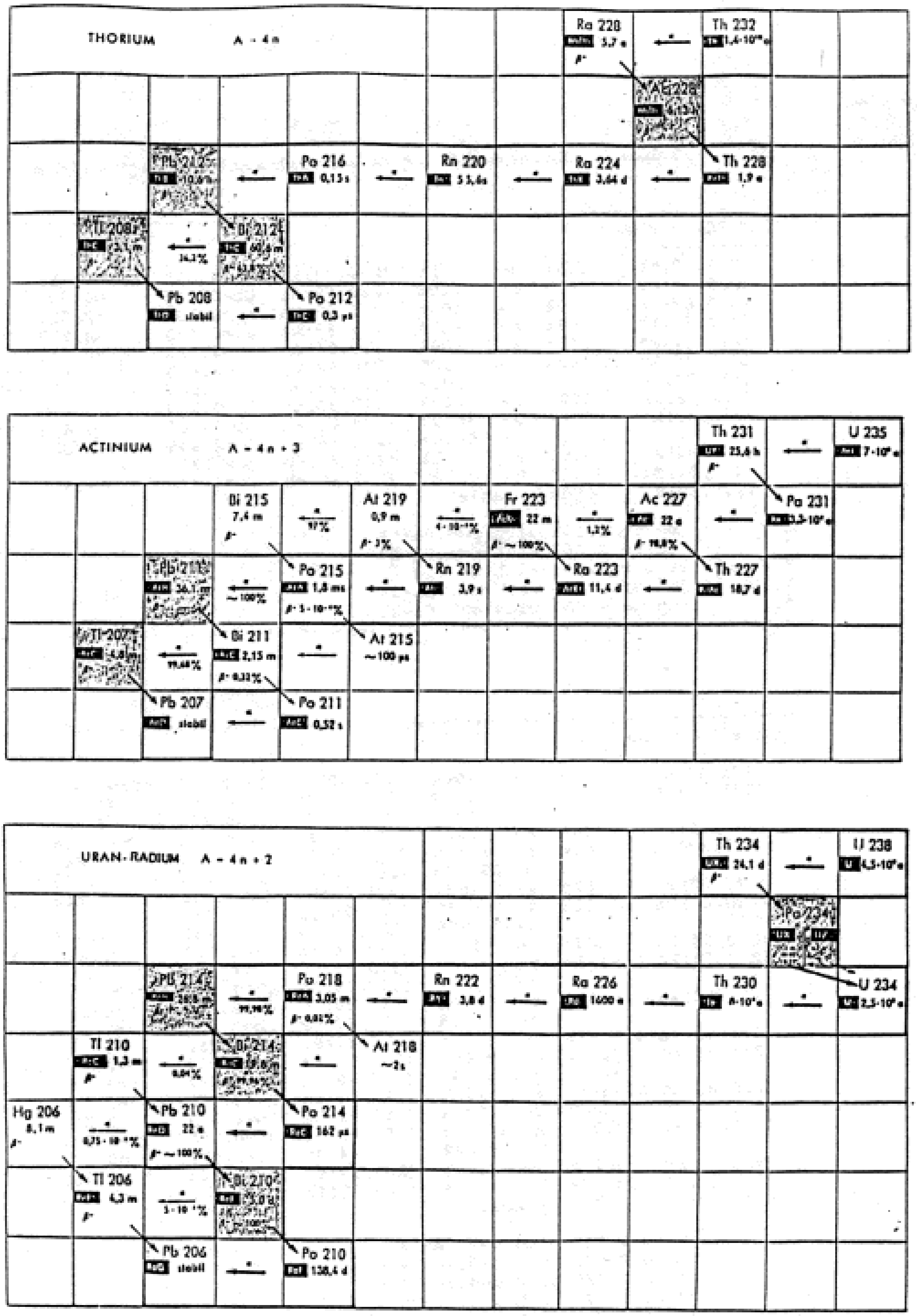


**Fig.1. Natural radioactive families.**
**Dangerous from the point of view to imitate double beta decay of 100-Mo isotopes are slightly darkened**

**Table 1. Characteristics of natural radioactive nuclides**

(232,90)Th chain:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (232,90)Th | 100% | a- | 100% | (228,88)Ra | Qa=4.081 | Eex=0.185 | | |
| (228,88)Ra | 100% | b- | 100% | (228,89)Ac | Qb=0.046 | =0.007 | | |
| (228,89)Ac | 100% | b- | 100% | (228,90)Th | =2.137 | =2.027 | | *** |
| (228,90)Th | 100% | a- | 100% | (224,88)Ra | Qa=5.520 | =0.290 | (1) | |
| (224,88)Ra | 100% | a- | 100% | (220,86)Rn | =5.789 | =0.241 | (2) | |
| (220,86)Rn | 100% | a- | 100% | (216,84)Po | =6.405 | =0. | (3) | |
| (216,84)Po | 100% | a- | 100% | (212,82)Pb | =6.907 | =0. | (4) | |
| (212,82)Pb | 100% | b- | 100% | (212,83)Bi | Qb=0.573 | =0.415 | | * |
| (212,83)Bi | 100% | a- | 36.0% | (208,81)Tl | Qa=6.207 | =0.493 | (5) | |
| | | b- | 64.0% | (212,84)Po | Qb=2.246 | =1.806 | | *** |
| (212,84)Po | 64.0% | a- | 100% | (208,82)Pb | Qa=8.954 | =? | | |
| (208,81)Tl | 36.0% | b- | 100% | (208,82)Pb | Qb=4.992 | =4.481 | | **** |
| (208,82)Pb | 100% | stable | | | | | | |

1: up to 0.993 with p=6.0e-5%
2: up to 0.663 with p=1.7e-2%
3: 99.9% and 0.550 with p=0.1%
4: 99.9979% and 0.805 with p=2.1e-3%
5: up to 0.803 with p=1.6e-2%

(235,92)U chain:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (235,92)U | 100% | a- | 100% | (231,90)Th | Qa=4.679 | Eex=0.452 | | |
| (231,90)Th | 100% | a- | ...% | (227,88)Ra | =4.202 | =? | | |
| | | b- | 100% | (231,91)Pa | Qb=0.389 | =0.352 | | * |
| (231,91)Pa | 100% | a- | 100% | (227,89)Ac | Qa=5.148 | =0.438 | (1) | |
| (227,89)Ac | 100% | a- | 1.38% | (223,87)Fr | =5.043 | =0.243 | (2) | |
| | | b- | 98.62% | (227,90)Th | Qb=0.044 | =0.025 | | |
| (227,90)Th | 98.62% | a- | 100% | (223,88)Ra | Qa=6.146 | =0.538 | (3) | |
| (223,87)Fr | 1.38% | a- | 0.005% | (219,85)At | =5.430 | =? | | |
| | | b- | 99.995% | (223,88)Ra | Qb=1.148 | =0.943 | | * |
| (223,88)Ra | ~100% | a- | 100% | (219,86)Rn | Qa=5.799 | =0.646 | (4) | |
| (219,85)At | 6.9e-5% | a- | 97% | (215,83)Bi | =6.390 | =? | | |
| | | b- | 3% | (219,86)Rn | Qb=1.700 | =? | | |
| (219,86)Rn | ~100% | a- | 100% | (215,84)Po | Qa=6.946 | =0.517 | (5) | |
| | | b- | ...% | (219,87)Fr | Qb=0.214 | =? | | |
| (215,83)Bi | 6.7e-5% | b- | 100% | (215,84)Po | =2.250 | =? | | |
| (215,84)Po | 100% | a- | ~100% | (211,82)Pb | Qa=7.527 | =0. | (6) | |
| | | b- | 2.3e-4% | (215,85)At | Qb=0.721 | =? | | |
| (215,85)At | 2.3e-4% | a- | 100% | (211,83)Bi | Qa=8.178 | =0. | (7) | |
| (211,82)Pb | ~100% | b- | 100% | (211,83)Bi | Qb=1.373 | =1.270 | | ** |
| (211,83)Bi | 100% | a- | 99.72% | (207,81)Tl | Qa=6.751 | =0.351 | | |
| | | b- | 0.28% | (211,84)Po | Qb=0.579 | =0. | | * |
| (211,84)Po | 0.28% | a- | 100% | (207,82)Pb | Qa=7.594 | =0.898 | | |
| (207,81)Tl | 99.72% | b- | 100% | (207,82)Pb | Qb=1.422 | =0.898 | | ** |
| (207,82)Pb | 100% | stable | | | | | | |

1: up to 0.657 with p=2.7e-2%
2: up to 0.601 with p=9.7e-2%
3: up to 1.025 with p=4.1e-2%
4: up to 0.873 with p=5.7e-2%
5: up to 1.055 with p=3.3e-2%
6: up to 0.445 with p=5.6e-2%
7: 99.95% and 0.405 with p=0.05%

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(238,92)U chain:
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   (238,92)U      100%  a-    100%  (234,90)Th   Qa=4.270   Eex=0.160
   (234,90)Th     100%  a-    ...%  (230,88)Ra     =3.630      =?
                        b-    100%  (234m,91)Pa  Qb=0.183      =0.133
   (234m,91)Pa    100%  g-   0.13%  (234,91)Pa   Qg=0.080      -
                        b-  99.87%  (234,92)U    Qb=2.287      =1.970        ***
   (234,91)Pa    0.13%  a-    ...%  (230,89)Ac   Qa=4.160      =?
                        b-    100%  (234,92)U    Qb=2.207      =2.143          *
   (234,92)U      100%  a-    100%  (230,90)Th   Qa=4.856      =0.174  (1)
   (230,90)Th     100%  a-    100%  (226,88)Ra     =4.771      =0.211  (2)
   (226,88)Ra     100%  a-    100%  (222,86)Rn     =4.871      =0.186  (3)
   (222,86)Rn     100%  a-    100%  (218,84)Po     =5.591      =0.     (4)
                        b-    ...%  (222,87)Fr   Qb=0.032      =?
   (218,84)Po     100%  a-99.982%   (214,82)Pb   Qa=6.115      =0.     (5)
                        b- 0.018%   (218,85)At   Qb=0.256      =?
   (218,85)At   0.018%  a-   99.9%  (214,83)Bi   Qa=6.883      =0.096
                        b-    0.1%  (218,86)Rn   Qb=2.887      =?
   (218,86)Rn 1.8e-5%   a-    100%  (214,84)Po   Qa=7.266      =0.609          *
   (214,82)Pb 99.982%   b-    100%  (214,83)Bi   Qb=1.024      =0.839
   (214,83)Bi    ~100%  a- 0.021%   (210,81)Tl   Qa=5.617      =0.581         **
                        b-99.979%   (214,84)Po   Qb=3.270      =2.729  (6) ****
   (214,84)Po 99.979%   a-    100%  (210,82)Pb   Qa=7.834      =0.     (7)
   (210,81)Tl  0.021%   b-    100%  (210,82)Pb   Qb=5.487      =?      (8)     *
   (210,82)Pb     100%  a-1.7e-6%   (206,80)Hg   Qa=3.792      =?
                        b-   ~100%  (210,83)Bi   Qb=0.063      =0.047
   (210,83)Bi    ~100%  a-1.3e-4%   (206,81)Tl   Qa=5.043      =0.305
                        b-   ~100%  (210,84)Po   Qb=1.161      =0.            **
   (210,84)Po    ~100%  a-    100%  (206,82)Pb   Qa=5.408      =0.     (9)
   (206,80)Hg 1.7e-6%   b-    100%  (206,81)Tl   Qb=1.313      =0.650
   (206,81)Tl 1.3e-4%   b-    100%  (206,82)Pb     =1.526      =0.     (10)
   (206,82)Pb     100%  stable
 1:         up to 0.678 with p=7.8e-5% and high probability to emit conv.el.
 2:         up to 0.446 with p=3.1e-2%
 3:         up to 0.636 with p=7.8e-2%
 4:         up to 0.675 with p=7.9e-2%
 5:  99.9989% and 0.837 with p=1.1e-3%
 6:         up to 3.184 with p=0.14%
 7:         up to 1.098 with p=0.01%
 8:         up to 5 MeV but probabilities are unknown
 9:   99.999% and 0.803 with p=1.0e-3%
10:         up to 1.167 with p=8.5e-2%
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  (40,19)K                 b-  89.3%  (40,20)Ca   Qb =1.312  Eex=0.
                           ec- 10.7%  (40,18)Ar   Qec=1.505     =1.461      **
  (60,27)Co                b-   100%  (60,28)Ni   Qb =2.824     =2.506     ***
  (137,55)Cs               b-   100%  (137,56)Ba     =1.173     =0.662      **
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a/b/g - alpha/beta/gamma
ec - electron capture
Q - energy release
Eex - maximum energy of excited level of daughter nucleus

...% - very low (and therefore undetermined) probability
~100% - more than 99.999%

**** - Q is more than 3 MeV, high probability for this mode of decay
*** - Q is near 2 MeV, high probability
** - Q is near 1 MeV, high probability
* - high Q but low (<1%) probability, or Q<0.6 MeV

## 2. Accepted approximations and simplifications

1. All probabilities for alpha and beta modes of decay, values of energy release, energies of excited nuclear levels and probabilities of transitions to lower levels are taken from [2] (it is the cause of some differences between fig. 1 and table 1). All energies are rounded to 0.001 Mev and probabilities - to 0.001%.

2. Decays of different nuclides in natural radioactive chains can be considered as independent because even for 3 nuclides with the most short half-lives (214-Po - 164 microsec, 215-At - 100 microsec and 212-Po - 0.3 microsec) the average time interval between decays of two successive nuclides is far out of range when they are considered as one event in NEMO detector (1-few ns [3]).

3. Halflives t of excited nuclear levels are considered as equal to 0; so, we have not delay in emission of gammas or/and conversion electrons or/and e+e- pairs in cascade of nuclear transitions in deexcitation process. It is excused by reasons that:

(1) for absolute majority of levels values t are unknown [2];

(2) in rare cases when t is known its value lies in the interval from 0.73 ps to 0.76 ns for nuclides listed in section 3. We have only three exceptions:
   - for level 0.662 MeV of 137-Ba created in beta decay of 137-Cs t=2.551 min. In NEMO detector beta decay of 137-Cs with energy 0.511 MeV and following deexcitation of 0.662 MeV level will be appeared as independent events; in this way they are considered in model for 137-Cs decay;
   - for level 0.266 MeV of 206-Tl created in alpha decay of 210-Bi t=3 ns but probability to occupy this level in decay of 210-Bi is equal to 5e-5%;
   - for level 1.421 MeV of 234-U t=33.5 microsec but probability to have the deexcitation of this level after beta decay of 234m-Pa is less than 8e-2%.

4. Angular correlations between gamma quanta in cascade of nuclear transitions are not considered; so, we have independent gammas (or/and e+e- pairs or/and conversion electrons) emitted isotropically in all directions. There are two reasons for this approximation:

(1) theoretical formulae for probability distribution to have different angles between two gammas depend on changes in spin and parity of initial and daughter states of nucleus and are well-established only in some cases (for transition 0+ -> 2+ for example). We have much more big variety of transition types;

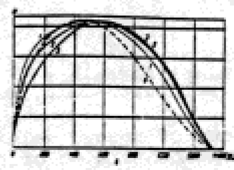
(2) information about spin and parity is known not for all levels, and in these cases we don't know the type of transition at all.

5. All beta decays are considered as allowed. Really the shape of energy spectrum of beta particles depends not only on energy release and atomic number of nucleus but also on differences in spin and parity of parent and daughter nuclei. These differences determine the type of beta decay: allowed, once-forbidden, once-forbidden-unique, twice-forbidden and so on. The examples of beta spectra of different degree forbiddenness for nuclei with Z=19, 30 and 90 are shown in fig. 2. We can list the following reasons to consider all beta decays as allowed:

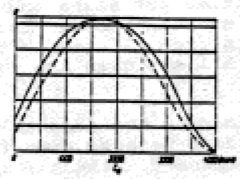
(1) differences in shapes are not so large to see them with the NEMO detector with current energy resolution (~16% at 1 MeV);

(2) theoretical formula for shape is simple only for allowed decay but corresponding formulae for forbidden decays become more and more complicated with rise of forbiddenness [4]; there are empirical coefficients often in these formulae what depends on nuclei and are not known for all isotopes;

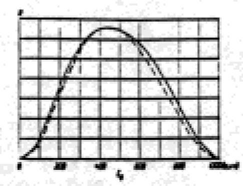
(3) we know spin and parity not for all excited levels of daughter nuclei occupied in beta decay of parent nuclei and therefore one can't determine the type of decay in all cases.



Theoretical shape of the once-forbidden β-spectrum for $Z =$ [illegible] (1) - first forbidden, (2) - second forbidden, (3) - third forbidden. The broken line represents the shape of the allowed β-spectrum. The curves are normalized with respect to the maximum of the ordinates.

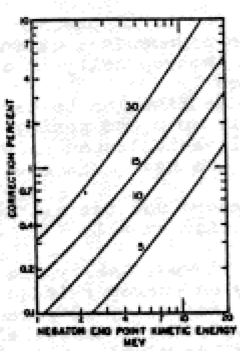


Comparison of the once-forbidden shape of first forbidden $\beta^-$-spectrum (unbroken line) with the allowed shape (broken line) for $Z =$ [illegible].



Comparison of the once-forbidden shape of first forbidden $\beta^+$-spectrum (unbroken line) with the allowed shape (broken line) for $Z =$ [illegible].

**Fig.2. Examples of beta spectra shapes for different degree of beta decay forbiddenness**



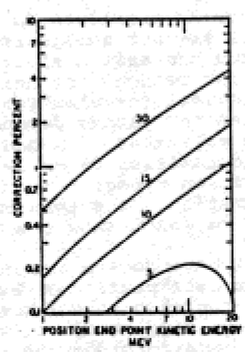




**Fig.3. The corrections to the intensity for electron or positron emission due to the finite nuclear size effects. The numbers on the curves are the Z-values**

6. Only Fermi correction factor to the shape of energy spectrum of beta particles is taken into account. Shape of energy spectrum in allowed decay is determined by expression

$$N(Wb)=Pb*Pn*Wb*Wn*S(Z,Wb),$$

where the product of two momenta P (of beta particle and neutrino) and two full energies W is so called kinematical factor and S describes the influence of Coulomb electric field of daughter nucleus on emitted charged particle (electron or positron), screening the nuclear charge by atomic electrons, radiative corrections, effects of nonzero nuclear size and so on - more than 15 corrections [5]. Main of them is influence of Coulomb field described by factor Fermi F(Z,Wb). The screening by the atomic electrons results effectively in the shift of spectrum on less than 14 keV for Z up to 100. Finite nuclear size effects also give not big corrections (fig. 3). So, our approximation S(Z,Wb)=F(Z,Wb) seems to be quite good for NEMO detector. For more details about calculation of F(Z,Wb) see section 6.

7. Full and practically exact nuclear schemes described in [2] - no approximations or simplifications - were used for models of decay for all nuclides dangerous from the point of view of imitating 100-Mo double beta decay (listed in section 3) except 234m-Pa. There were some cases when sum of probabilities of decay or transition to lower levels was not equal to 100% exactly. For example, sum of probabilities of beta decay 214-Bi to 214-Po excited levels up to 3.184 MeV is equal to 100.9225% according to [2] instead of 100%; sum of probabilities for transitions to lower levels from level 3.709 MeV of 208-Pb created in 208-Tl beta decay is equal to 100.1%. Slight correction of probabilities were made in these cases to have the 100% sum exactly.

Such approach - without simplifications - gives quite sophisticated models sometimes:

(1) for beta decay of 208-Tl possibilities to occupy 14 excited levels of 208-Pb are taken into account; in following deexcitation process 25 different gamma quanta can be emitted;
(2) for decay of 214-Bi 48 levels of 214-Po (214-Bi beta decay), 3 levels of 210-Tl (214-Bi alpha decay) and 105 different gamma quanta are included in the model;
(3) for beta decay of 228-Ac model contains 44 excited levels of 228-Th and 166 different gamma quanta.

Only in the case of 234m-Pa some simplifications were made:

(1) isomeric transition 234m-Pa to 234-Pa is not considered (its probability is equal 0.13% and scheme of 234-Pa further decay to 234-U is not very well determined [2]);
(2) decays of 234m-Pa to excited levels of 234-U with energies greater than 1.045 MeV are not considered (probability is equal to 0.17%).

So, current decay model of 234m-Pa describes 99.7% of all possibilities that is not so bad.

It should be noted that joint beta-alpha decays were not considered also: for decay 212-Bi -> 208-Pb probability of this type transition is equal 1.4e-2%, for 214-Bi -> 210-Pb - 2.8e-3%.

8. For each transition from excited nuclear level to one of lower levels possibility of all of three concurent processes are taken into account in models:

(1) to emit gamma quantum;
(2) to emit the conversion electron instead of gamma;
(3) to emit the electron-positron pair; in this case energy of transition must be greater than 1.022 MeV.

Such an approach allows automatically to consider the emission not only of one but of two or more conversion electrons and/or e+e- pairs - of course, with corresponding probabilities.

9. Coefficient of conversion of gamma quanta to conversion electrons (i.e., number of emitted electrons to the number of gammas) depends:

(1) on energy of transition E;
(2) on atomic number of nucleus Z;
(3) on type of transition (electrical E or magnetic M) and emission

multipolarity L;
(4) on what subshell emitted electron is belonged.

For Z~80-90 (our region of interest) a change in type of transition (M instead of E) increases the conversion coefficient by order of magnitude; an increase in L by 1 results in rise of coefficient by 2-3 times. So, one must be very attentive when defining these coefficients.

It is useful to give some examples of concrete coefficient values for most frequent types of transition (it is the sum for all subshells):

| | | M1 | M2 | E1 | E2 |
|---|---|---|---|---|---|
| Z=80 | E=0.5 MeV | 9.5e-2 | 2.8e-1 | 9.0e-3 | 2.6e-2 |
| | 1.0 | 1.6e-2 | 4.0e-2 | 2.3e-3 | 6.0e-3 |
| | 1.5 | 6.0e-3 | 1.4e-2 | 1.2e-3 | 2.8e-3 |
| Z=90 | E=0.5 MeV | 2.2e-1 | 6.0e-1 | 1.3e-2 | 4.3e-2 |
| | 1.0 | 3.5e-2 | 8.0e-2 | 3.5e-3 | 1.0e-2 |
| | 1.5 | 1.2e-2 | 2.8e-2 | 1.8e-3 | 4.5e-3 |

It should be noted that sometimes (for E0 transition) only emission of conversion electron but no gamma is allowed.

Following procedure was accepted to determine the electron conversion coefficients for all transitions in models:

(1) type and multipolarity of transition were defined either from direct information in [2] or - when such information was absent - according to changes in spin and parity of nucleus in result of transition;
(2) for given Z, E, type and multipolarity theoretical conversion coefficient was determined due to graphs in [6];
(3) experimental values [2] known for most intensive transitions were used to correct the totality of theoretical values when it was needed;
(4) in cases when information about spin and parity of levels was absent [2] minimal possible value for given Z and E was accepted, i.e., E1 type of transition was assumed to determine the electron conversion coefficient.

As an example for point (3), for transition with energy release 2.615 MeV in decay 208-Tl -> 208-Pb theoretical electron conversion coefficient is equal 2.4e-3 (E3 type of transition) [6] and experimental value - 2e-3 with uncertainty 1e-3 [2].

10. Only conversion from K shell was assumed in all cases (absolute majority) when transition energy was sufficient, i.e., when E(transition) > E(binding energy of K shell electron). In the same time full conversion coefficient for all shells (K+L+...) [6] was used to calculate the probability of electron conversion. It is not very rough simplification because typical value of probability for conversion from L shell is 20% from that of K shell; for M shell it is still less.

When transition energy was unsufficient for K conversion, L conversion was assumed.

As a result of conversion we have always the conversion electron with energy E(conversion electron)=E(transition)-E(shell binding) and gamma quantum with E(shell binding) (but no Auger electron).

11. If energy released in transition is greater 1.022 MeV electron-positron pair can be emitted instead of gamma quantum. Coefficient of conversion to pair (i.e., number of emitted pairs to the number of gammas) depends on the same factors as electron conversion coefficient (see point 9).

Typical values of pair conversion coefficient for most frequent types of transition (dependence on Z iz quite weak):

| | M1 | M2 | E1 | E2 |
|---|---|---|---|---|
| E=1.5 MeV | 0.9e-4 | 0.5e-4 | 0.5e-4 | 0.3e-4 |
| 2.0 | 2.3e-4 | 1.4e-4 | 3.6e-4 | 2.0e-4 |
| 2.5 | 4.5e-4 | 2.8e-4 | 9.3e-4 | 5.6e-4 |

Procedure used to determine the pair conversion coefficients for all transitions in models was the same as for electron conversion coefficients (point 9). Graphs from [7] were used as source of theoretical values (figures 53 and 54 on page 368 of [7] were enlarged with this aim), and it was possible to find in [2] experimental values for some intensive transitions for correction. As an example of agreement between theoretical and experimental values, for transition 2.615 MeV in decay of 208-Tl mentioned in point 9 theoretical pair conversion coefficient [7] is equal 4.1e-4 and experimental value [2] - 4.3e-4 with uncertainty 0.7e-4.

12. Sum of electron and positron energies in emitted pair is equal to E(transition)-1.022 MeV. Current subroutine for pair creation assumes that:
(1) e+ and e- have equal energies;
(2) they are emitted in the same direction.
Although quite rough, these assumptions have a physical sense (for more details see section 7).

13. Future possible improvements to description of pair creation and/or beta decay will change mainly the subroutines for simulation of corresponding processes but no the models of nuclides decay.

---

## 3. Models of decay

---

Following information is given for models:
(1) simplifications - if they were used;
(2) modes of decay (alpha/beta), their probabilities and energy release;
(3) description of decay and deexcitation process;
(4) binding shell energies of electron in daughter nucleus.

Following abbreviations were used:
- beta(0.609) - beta particle with maximal kinetic energy 0.609 MeV;
- conv.el.(0.609) - conversion electron with kinetic energy 0.609 MeV;
- gamma(0.609) - gamma quantum with energy 0.609 MeV;
- T(1.283: g=1. ce=8.5e-3 p=0.5e-4) - nuclear transition with energy release 1.283 MeV and relative weights to emit: gamma quantum = 1, conversion electron = 8.5e-3 and pair = 0.5e-4. Weights ce and p are electron and pair conversion coefficients (see section 2) and probabilities of emission are equal:
  - for gamma - 1./(1.+ce+p);
  - for conversion electron - ce/(1.+ce+p);
  - for pair - p/(1.+ce+p).

The line:

```
0.052%  beta(0.511) -> level 4481 keV of Pb208
```

means that with probability 0.052% beta particle with maximal kinetic energy 0.511 MeV is emitted and level 4481 keV of daughter nucleus is occupied after that.

The lines:

```
level 3475  64%  T(0.860:  g=1.  ce=2.9e-2  p=0.)  -> level 2615
            36%  T(0.277:  g=1.  ce=0.45    p=0.)  ->       3198
```

means that level 3475 keV has two possibilities to be deexcited:
- with probability 64% - to radiate the energy 0.860 MeV (in the form of gamma or conversion electron or pair with corresponding probabilities) and after that level 2615 keV is occupied;
- with probability 36% - to emit the energy 0.277 MeV and after that level 3198 keV is occupied.

The arrow to level 0 means that deexcitation process is finished.

```
--------------------------------------------------------------------------------
                              Model of K40 decay
--------------------------------------------------------------------------------
89.300%  betaminus decay to ground level of Ca40            Q(beta-)=1.312:
         beta(1.312 with Z of daughter nucleus = +20)  -> level 0 kev of Ca40

10.699%  electron capture to level 1461 kev of Ar40         Q(ec)    =1.505:
         T(1.461:  g=1.  ce=3.0e-5  p=7.0e-5)  -> level 0 kev of Ar40

 0.001%  betaplus decay to ground level of Ar40             Q(beta+)=0.483:
         beta(0.483 with Z of daughter nucleus = -18)  -> level 0 kev of Ar40

Binding energy of K-shell Ar electron = 0.003
```

```
-------------------------------------------------------------------------------
                              Model of Co60 decay
-------------------------------------------------------------------------------

   100% beta decay to Ni60     Q(beta)=2.824

99.880%  beta(0.318) -> level 2506 keV of Ni60
 0.120%  beta(1.491) ->        1333

level 2506   99.992%  T(1.173:  g=1.  ce=1.5e-4  p=0.1e-4) -> level 1333
              0.008%  T(0.347:  g=1.  ce=6.5e-3  p=0.    ) ->       2159

      2159       13%  T(2.159:  g=1.  ce=5.5e-5  p=3.5e-4) ->          0
                 87%  T(0.826:  g=1.  ce=3.5e-4  p=0.    ) ->       1333

      1333            T(1.333:  g=1.  ce=1.1e-4  p=0.3e-4) ->          0

Binding energy of K-shell Ni electron = 0.008
```

---

## Model of Cs137 decay

---

Excited level 0.662 MeV of Ba137 has very big half-life (2.551 min) as compared with timing in NEMO detector (1-few ns). So, for NEMO detector beta decay with energy 0.511 MeV and following after that transition 0.662 MeV are regarded as independent.

100% beta decay to Ba137     Q(beta)=1.173

```
48.613%  beta(0.511)
48.613%  T(0.662:  g=1.  ce=1.1e-1  p=0.)
 2.774%  beta(1.173)
```

Binding energy of K-shell Ba electron = 0.037

```
---------------------------------------------------------------------------------
                              Model of Tl207 decay
---------------------------------------------------------------------------------

  100% beta decay to Pb207   Q(beta)=1.422

 0.24%  beta(0.524) -> level 898 keV of Pb207
99.76%  beta(1.422) ->          0

level 898  99.4% T(0.898:  g=1.  ce=2.0e-2  p=0.) -> level   0
            0.6% T(0.328:  g=1.  ce=2.8e-1  p=0.) ->       570

      570        T(0.570:  g=1.  ce=2.0e-2  p=0.) ->         0

Binding energy of K-shell Pb electron = 0.088
```

```
------------------------------------------------------------------------
                         Model of Tl208 decay
------------------------------------------------------------------------

  100% beta decay to Pb208    Q(beta)=4.992

 0.052%  beta(0.511) -> level 4481 keV of Pb208
 0.017%  beta(0.609) ->       4383
 0.043%  beta(0.634) ->       4358
 0.082%  beta(0.696) ->       4296
 0.220%  beta(0.812) ->       4180
 0.160%  beta(0.867) ->       4125
 0.007%  beta(0.996) ->       3996
 3.100%  beta(1.031) ->       3961
 0.040%  beta(1.046) ->       3946
 0.570%  beta(1.072) ->       3920
22.800%  beta(1.283) ->       3709
21.700%  beta(1.517) ->       3475
51.179%  beta(1.794) ->       3198
 0.030%  beta(2.377) ->       2615

level 4481        T(1.283:  g=1.  ce=8.5e-3  p=0.5e-4) -> level 3198
      4383        T(1.185:  g=1.  ce=1.1e-2  p=0.3e-4) ->       3198
      4358    5%  T(1.744:  g=1.  ce=4.0e-3  p=1.5e-4) ->       2615
             25%  T(1.161:  g=1.  ce=1.1e-2  p=0.3e-4) ->       3198
             70%  T(0.883:  g=1.  ce=2.2e-2  p=0.    ) ->       3475
      4296   50%  T(0.821:  g=1.  ce=2.6e-2  p=0.    ) ->       3475
             50%  T(0.589:  g=1.  ce=6.5e-2  p=0.    ) ->       3709
      4180   90%  T(0.983:  g=1.  ce=1.7e-2  p=0.    ) ->       3198
             10%  T(0.705:  g=1.  ce=4.0e-2  p=0.    ) ->       3475
      4125   78%  T(0.928:  g=1.  ce=1.9e-2  p=0.    ) ->       3198
             22%  T(0.650:  g=1.  ce=5.0e-2  p=0.    ) ->       3475
      3996        T(1.381:  g=1.  ce=8.0e-3  p=0.6e-4) ->       2615
      3961   66%  T(0.763:  g=1.  ce=3.1e-2  p=0.    ) ->       3198
              2%  T(0.486:  g=1.  ce=2.1e-1  p=0.    ) ->       3475
             32%  T(0.253:  g=1.  ce=6.9e-1  p=0.    ) ->       3709
      3946        T(0.749:  g=1.  ce=3.5e-2  p=0.    ) ->       3198
      3920   54%  T(0.722:  g=1.  ce=3.6e-2  p=0.    ) ->       3198
             46%  T(0.211:  g=1.  ce=1.     p=0.    ) ->       3709
      3709  1.7%  T(1.094:  g=1.  ce=0.5e-2  p=0.    ) ->       2615
           96.9%  T(0.511:  g=1.  ce=0.1     p=0.    ) ->       3198
            1.4%  T(0.233:  g=1.  ce=0.8     p=0.    ) ->       3475
      3475   64%  T(0.860:  g=1.  ce=2.9e-2  p=0.    ) ->       2615
             36%  T(0.277:  g=1.  ce=0.45    p=0.    ) ->       3198
      3198        T(0.583:  g=1.  ce=2.0e-2  p=0.    ) ->       2615
      2615        T(2.615:  g=1.  ce=2.4e-3  p=4.3e-4) ->          0

Binding energy of K-shell Pb electron = 0.088
```

```
--------------------------------------------------------------------------------
                              Model of Bi210 decay
--------------------------------------------------------------------------------

 0.00013%  alpha decay to Tl206     Q(alpha)=5.043
99.99987%   beta decay to Po210     Q(beta) =1.161

alpha:  60%  alpha(4.650) -> level 305 keV of Tl206
        40%  alpha(4.687) ->       266

      level 305  T(0.305:  g=1.  ce=3.5e-1  p=0.) -> level 0

            266  T(0.266:  g=1.  ce=1.5e-1  p=0.) ->       0

      Binding energy of K-shell Tl electron = 0.086

beta:  100%  beta(1.161) -> level 0 keV of Po210
```

```
-------------------------------------------------------------------------------
                              Model of Pb211 decay
-------------------------------------------------------------------------------

  100% beta decay to Bi211     Q(beta)=1.373

  0.01%    beta(0.103) -> level 1270 keV of Bi211
  0.02%    beta(0.177) ->       1196
  0.60%    beta(0.264) ->       1109
  0.04%    beta(0.293) ->       1080
  5.30%    beta(0.541) ->        832
  0.30%    beta(0.607) ->        766
  1.80%    beta(0.968) ->        405
 91.93%    beta(1.373) ->          0

level 1270     50%  T(1.270: g=1.  ce=8.7e-3  p=0.5e-4) -> level    0
               45%  T(0.866: g=1.  ce=2.3e-2  p=0.     ) ->        405
                5%  T(0.504: g=1.  ce=9.3e-2  p=0.     ) ->        766

      1196     15%  T(1.196: g=1.  ce=1.0e-2  p=0.2e-4) ->           0
               12%  T(0.429: g=1.  ce=1.5e-1  p=0.     ) ->        766
               73%  T(0.244: g=1.  ce=6.7e-1  p=0.     ) ->        951

      1109   15.9%  T(1.109: g=1.  ce=1.2e-2  p=0.1e-4) ->           0
             71.3%  T(0.705: g=1.  ce=3.9e-2  p=0.     ) ->        405
              3.0%  T(0.343: g=1.  ce=2.6e-1  p=0.     ) ->        766
              7.9%  T(0.274: g=1.  ce=5.0e-1  p=0.     ) ->        832
              1.9%  T(0.095: g=1.  ce=4.0e+0  p=0.     ) ->       1014

      1080     10%  T(1.080: g=1.  ce=1.3e-2  p=0.1e-4) ->           0
               13%  T(0.675: g=1.  ce=4.3e-2  p=0.     ) ->        405
               25%  T(0.314: g=1.  ce=3.5e-1  p=0.     ) ->        766
               52%  T(0.244: g=1.  ce=6.7e-1  p=0.     ) ->        832

      1014     37%  T(1.014: g=1.  ce=1.7e-2  p=0.     ) ->          0
               63%  T(0.609: g=1.  ce=5.7e-2  p=0.     ) ->        405

       951          T(0.951: g=1.  ce=1.8e-2  p=0.     ) ->          0

       832   65.8%  T(0.832: g=1.  ce=2.8e-2  p=0.     ) ->          0
             33.0%  T(0.427: g=1.  ce=1.6e-1  p=0.     ) ->        405
              1.2%  T(0.066: g=1.  ce=5.3e+0  p=0.     ) ->        766

       766          T(0.766: g=1.  ce=3.9e-2  p=0.     ) ->          0

       405          T(0.405: g=1.  ce=1.0e-1  p=0.     ) ->          0

Binding energy of K-shell Bi electron = 0.091
                  L-shell             = 0.016
```

```
--------------------------------------------------------------------------------
                            Model of Bi212 decay
--------------------------------------------------------------------------------

Beta-alpha decay to Pb208 is not considered (p=0.014%).

36%  alpha decay to Tl208        Q(alpha)=6.207
64%   beta decay to Po212        Q(beta) =2.246

alpha:  1.10%  alpha(5.607) -> level 493 keV of Tl208
        0.15%  alpha(5.626) ->       473
        1.67%  alpha(5.769) ->       328
       69.88%  alpha(6.051) ->        40
       27.20%  alph (6.090) ->         0

      level 493   5   T(0.493:  g=1.  ce=2.8e-2  p=0.) -> level   0
                 94%  T(0.453:  g=1.  ce=0.18    p=0.) ->        40
                  1%  T(0.164:  g=1.  ce=0.75    p=0.) ->       328

            473  68%  T(0.474:  g=1.  ce=0.14    p=0.) ->         0
                 19%  T(0.434:  g=1.  ce=0.14    p=0.) ->        40
                 13%  T(0.145:  g=1.  ce=2.8     p=0.) ->       328

            328  29%  T(0.328:  g=1.  ce=0.33    p=0.) ->         0
                 71%  T(0.288:  g=1.  ce=0.53    p=0.) ->        40

             40       T(0.040:  g=1.  ce=22.55   p=0.) ->         0

      Binding energy of K-shell Tl electron = 0.086
                        L-shell             = 0.015

beta:   0.660%  beta(0.440) -> level 1806 keV of Po212
        0.027%  beta(0.445) ->       1801
        0.250%  beta(0.566) ->       1680
        1.900%  beta(0.625) ->       1621
        1.500%  beta(0.733) ->       1513
        4.400%  beta(1.519) ->        727
       55.263%  beta(2.246) ->          0

      level 1806  17% T(1.806:  g=1.  ce=2.6e-2  p=1.7e-4) -> level   0
                  83% T(1.079:  g=1.  ce=2.0e-2  p=0.    ) ->       727

            1801  35% conv.el.(1.708) + gamma(0.093)        ->         0
                  65% T(1.074:  g=1.  ce=7.0e-3  p=0.    ) ->       727

            1680  28% T(1.680:  g=1.  ce=2.8e-3  p=1.0e-4) ->         0
                  72% T(0.952:  g=1.  ce=4.5e-2  p=0.    ) ->       727

            1621  80% T(1.621:  g=1.  ce=7.0e-3  p=1.2e-4) ->         0
                  20% T(0.893:  g=1.  ce=4.5e-2  p=0.    ) ->       727

            1513  22% T(1.513:  g=1.  ce=3.5e-3  p=0.7e-4) ->         0
                  78% T(0.786:  g=1.  ce=4.1e-2  p=0.    ) ->       727

             727      T(0.727:  g=1.  ce=1.7e-2  p=0.    ) ->         0

      Binding energy of K-shell Po electron = 0.093
```

```
-----------------------------------------------------------------------------
                         Model of Pb212 decay
-----------------------------------------------------------------------------
100% beta decay to Bi212   Q(beta)=0.573

  5%  beta(0.158) -> level 415 keV of Bi212
 83%  beta(0.334) ->       239
 12%  beta(0.573) ->         0

level 415    0.5%  T(0.415:  g=1.  ce=0.24  p=0.) -> level  0
            98.0%  T(0.300:  g=1.  ce=0.55  p=0.) ->       115
             1.5%  T(0.177:  g=1.  ce=2.4   p=0.) ->       239

      239          T(0.239:  g=1.  ce=1.1   p=0.) ->         0

      115          T(0.115:  g=1.  ce=8.0   p=0.) ->         0

Binding energy of K-shell Bi electron = 0.091
```

```
------------------------------------------------------------------------------
                              Model of Bi214 decay
------------------------------------------------------------------------------

Beta-alpha decay to Pb210 is not considered (p=2.8e-3%).

 0.021%  alpha to Tl210      Q(alpha)=5.617
99.979%   beta to Po214      Q(beta) =3.270

alpha:
------
 5.9%  alpha(5.268) -> level 253 keV of Tl210
54.5%  alpha(5.448) ->          63
39.6%  alpha(5.512) ->           0

level 253   T(0.19    g=1.  ce=8.0e-2  p=0.) -> level 63

      63    T(0.06    g=1.  ce=2.8e-1  p=0.) ->        0

Binding energy of K-shell Tl electron = 0.086
                  L-shell             = 0.015

beta:
-----
 0.002%  beta(0.086) -> level 3184 keV of Po214
 0.002%  beta(0.127) ->       3143
 0.004%  beta(0.188) ->       3082
 0.023%  beta(0.216) ->       3054
 0.009%  beta(0.270) ->       3000
 0.015%  beta(0.291) ->       2979
 0.023%  beta(0.329) ->       2941
 0.016%  beta(0.348) ->       2922
 0.006%  beta(0.376) ->       2894
 0.009%  beta(0.390) ->       2880
 0.003%  beta(0.443) ->       2827
 0.006%  beta(0.484) ->       2786
 0.026%  beta(0.500) ->       2770
 0.410%  beta(0.541) ->       2729
 0.210%  beta(0.551) ->       2719
 0.060%  beta(0.571) ->       2699
 0.120%  beta(0.575) ->       2695
 0.070%  beta(0.608) ->       2662
 0.020%  beta(0.639) ->       2631
 0.051%  beta(0.725) ->       2545
 0.110%  beta(0.762) ->       2508
 0.200%  beta(0.764) ->       2506
 1.000%  beta(0.787) ->       2483
 2.800%  beta(0.822) ->       2448
 0.018%  beta(0.847) ->       2423
 0.560%  beta(0.977) ->       2293
 0.140%  beta(1.003) ->       2267
 0.330%  beta(1.061) ->       2209
 5.500%  beta(1.066) ->       2204
 0.880%  beta(1.077) ->       2193
 0.430%  beta(1.122) ->       2148
 4.277%  beta(1.151) ->       2119
 0.140%  beta(1.181) ->       2089
 2.500%  beta(1.253) ->       2017
 1.500%  beta(1.259) ->       2011
 1.200%  beta(1.275) ->       1995
 1.600%  beta(1.380) ->       1890
 8.150%  beta(1.423) ->       1847
17.800%  beta(1.505) ->       1765
 0.260%  beta(1.527) ->       1743
17.800%  beta(1.540) ->       1730
```

```
 0.960%  beta(1.609)  ->        1661
 3.300%  beta(1.727)  ->        1543
 1.000%  beta(1.855)  ->        1415
 7.450%  beta(1.892)  ->        1378
 0.210%  beta(1.995)  ->        1275
 1.000%  beta(2.661)  ->         609
17.800%  beta(3.270)  ->           0

level 3184          T(3.184:  g=1.  ce=4.0e-4  p=8.0e-4)  -> level     0

      3143          T(3.143:  g=1.  ce=4.0e-4  p=8.0e-4)  ->            0

      3082          T(3.082:  g=1.  ce=4.2e-4  p=8.0e-4)  ->            0

      3054          T(3.054:  g=1.  ce=4.3e-4  p=8.0e-4)  ->            0

      3000          T(3.000:  g=1.  ce=4.5e-4  p=8.0e-4)  ->            0

      2979          T(2.979:  g=1.  ce=4.5e-4  p=8.0e-4)  ->            0

      2941     7%   T(2.940:  g=1.  ce=4.5e-4  p=8.0e-4)  ->            0
              93%   T(2.331:  g=1.  ce=7.0e-4  p=4.6e-4)  ->          609

      2922          T(2.922:  g=1.  ce=4.5e-4  p=8.0e-4)  ->            0

      2894          T(2.894:  g=1.  ce=5.0e-4  p=7.3e-4)  ->            0

      2880          T(2.880:  g=1.  ce=5.0e-4  p=7.3e-4)  ->            0

      2827          T(2.827:  g=1.  ce=5.0e-4  p=7.3e-4)  ->            0

      2786          T(2.786:  g=1.  ce=5.0e-4  p=7.3e-4)  ->            0

      2770          T(2.770:  g=1.  ce=5.0e-4  p=7.3e-4)  ->            0

      2729     7%   T(1.067:  g=1.  ce=1.1e-3  p=0.     )  ->         1661
              93%   T(0.964:  g=1.  ce=3.0e-3  p=0.     )  ->         1765

      2719   0.8%   T(2.719:  g=1.  ce=6.0e-4  p=7.0e-4)  ->            0
            41.7%   T(2.110:  g=1.  ce=8.5e-4  p=2.8e-4)  ->          609
            57.5%   T(1.304:  g=1.  ce=2.6e-2  p=0.1e-4)  ->         1415

      2699     5%   T(2.699:  g=1.  ce=6.0e-4  p=7.3e-4)  ->            0
              95%   T(2.090:  g=1.  ce=3.5e-2  p=2.8e-4)  ->          609

      2695  27.3%   T(2.695:  g=1.  ce=6.0e-4  p=7.3e-4)  ->            0
            72.7%   T(1.317:  g=1.  ce=1.8e-3  p=0.6e-4)  ->         1378

      2662   0.4%   T(2.662:  g=1.  ce=6.0e-4  p=7.0e-4)  ->            0
            99.6%   T(2.053:  g=1.  ce=9.0e-4  p=2.4e-4)  ->          609

      2631     5%   T(2.631:  g=1.  ce=6.0e-4  p=6.4e-4)  ->            0
              95%   T(2.022:  g=1.  ce=9.0e-4  p=2.4e-4)  ->          609

      2545          T(1.936:  g=1.  ce=9.5e-4  p=2.8e-4)  ->          609

      2508    58%   T(1.899:  g=1.  ce=9.5e-4  p=2.8e-4)  ->          609
              42%   T(1.131:  g=1.  ce=2.2e-3  p=0.1e-4)  ->         1378

      2506   2.9%   T(2.506:  g=1.  ce=6.5e-4  p=5.6e-4)  ->            0
            87.1%   T(1.896:  g=1.  ce=9.5e-4  p=2.0e-4)  ->          609
            10.0%   T(1.230:  g=1.  ce=1.9e-3  p=0.3e-4)  ->         1275

      2483   0.2%   T(2.483:  g=1.  ce=1.4e-3  p=4.4e-4)  ->            0
            20.0%   T(1.873:  g=1.  ce=5.0e-3  p=2.0e-4)  ->          609
            44.2%   T(1.208:  g=1.  ce=2.6e-2  p=0.2e-4)  ->         1275
```

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 7.6% | T(1.105: | g=1. | ce=1.0e-2 | p=0.1e-4) | -> | 1378 |
| | 14.0% | T(0.821: | g=1. | ce=2.7e-2 | p=0. ) | -> | 1661 |
| | 12.0% | T(0.753: | g=1. | ce=1.0e-2 | p=0. ) | -> | 1730 |
| 2448 | 55.8% | T(2.448: | g=1. | ce=3.0e-4 | p=8.5e-4) | -> | 0 |
| | 14.2% | T(1.838: | g=1. | ce=1.0e-3 | p=1.8e-4) | -> | 609 |
| | 2.0% | T(1.173: | g=1. | ce=6.0e-3 | p=0.3e-4) | -> | 1275 |
| | 10.1% | T(1.070: | g=1. | ce=2.6e-3 | p=0. ) | -> | 1378 |
| | 3.0% | T(1.032: | g=1. | ce=2.6e-3 | p=0. ) | -> | 1415 |
| | 3.9% | T(0.904: | g=1. | ce=3.5e-3 | p=0. ) | -> | 1543 |
| | 11.1% | T(0.786: | g=1. | ce=3.8e-2 | p=0. ) | -> | 1661 |
| 2423 | 33% | T(2.424: | g=1. | ce=6.5e-4 | p=4.3e-4) | -> | 0 |
| | 67% | T(1.814: | g=1. | ce=1.0e-3 | p=1.8e-4) | -> | 609 |
| 2293 | 58% | T(2.293: | g=1. | ce=8.0e-3 | p=3.6e-4) | -> | 0 |
| | 42% | T(1.684: | g=1. | ce=1.2e-3 | p=1.3e-4) | -> | 609 |
| 2267 | 13% | T(2.267: | g=1. | ce=7.0e-4 | p=3.4e-4) | -> | 0 |
| | 54% | T(1.657: | g=1. | ce=1.2e-3 | p=1.2e-4) | -> | 609 |
| | 33% | T(0.723: | g=1. | ce=5.0e-3 | p=0. ) | -> | 1543 |
| 2209 | | T(1.599: | g=1. | ce=1.2e-3 | p=1.0e-4) | -> | 609 |
| 2204 | 89.2% | T(2.204: | g=1. | ce=2.8e-3 | p=3.1e-4) | -> | 0 |
| | 4.7% | T(1.595: | g=1. | ce=7.5e-3 | p=1.1e-4) | -> | 609 |
| | 1.7% | T(0.826: | g=1. | ce=2.8e-2 | p=0. ) | -> | 1378 |
| | 0.8% | T(0.661: | g=1. | ce=7.0e-2 | p=0. ) | -> | 1543 |
| | 1.5% | T(0.543: | g=1. | ce=1.2e-1 | p=0. ) | -> | 1661 |
| | 2.1% | T(0.474: | g=1. | ce=1.6e-1 | p=0. ) | -> | 1730 |
| 2193 | 6.9% | T(2.193: | g=1. | ce=1.8e-3 | p=3.0e-4) | -> | 0 |
| | 81.2% | T(1.583: | g=1. | ce=7.8e-3 | p=1.1e-4) | -> | 609 |
| | 5.0% | T(0.815: | g=1. | ce=4.0e-2 | p=0. ) | -> | 1378 |
| | 6.9% | T(0.649: | g=1. | ce=7.0e-2 | p=0. ) | -> | 1543 |
| 2148 | 4% | T(2.148: | g=1. | ce=1.0e-1 | p=2.9e-4) | -> | 0 |
| | 96% | T(1.539: | g=1. | ce=1.3e-3 | p=1.0e-4) | -> | 609 |
| 2119 | 27.7% | T(2.119: | g=1. | ce=2.9e-3 | p=2.9e-4) | -> | 0 |
| | 50.5% | T(1.509: | g=1. | ce=9.0e-3 | p=0.9e-4) | -> | 609 |
| | 0.9% | T(0.741: | g=1. | ce=5.0e-2 | p=0. ) | -> | 1378 |
| | 10.9% | T(0.703: | g=1. | ce=4.0e-2 | p=0. ) | -> | 1415 |
| | 10.0% | T(0.389: | g=1. | ce=2.6e-1 | p=0. ) | -> | 1730 |
| 2089 | 48% | T(1.479: | g=1. | ce=1.5e-3 | p=0.9e-4) | -> | 609 |
| | 52% | T(0.711: | g=1. | ce=5.5e-3 | p=0. ) | -> | 1378 |
| 2017 | 0.17% | conv.el.(1.923) | + gamma(0.093) | | | -> | 0 |
| | 99.83% | T(1.408: | g=1. | ce=4.0e-3 | p=0.2e-4) | -> | 609 |
| 2011 | 3.3% | T(2.011: | g=1. | ce=9.0e-4 | p=2.4e-4) | -> | 0 |
| | 92.7% | T(1.402: | g=1. | ce=1.6e-3 | p=0.8e-4) | -> | 609 |
| | 4.0% | T(0.633: | g=1. | ce=6.5e-3 | p=0. ) | -> | 1378 |
| 1995 | 66% | T(1.385: | g=1. | ce=1.6e-3 | p=0.3e-4) | -> | 609 |
| | 34% | T(0.720: | g=1. | ce=1.7e-2 | p=0. ) | -> | 1275 |
| 1890 | 6% | T(1.890: | g=1. | ce=2.4e-3 | p=2.0e-4) | -> | 0 |
| | 94% | T(1.281: | g=1. | ce=1.0e-2 | p=0.5e-4) | -> | 609 |
| 1847 | 25.5% | T(1.847: | g=1. | ce=3.4e-3 | p=1.8e-4) | -> | 0 |
| | 71.9% | T(1.238: | g=1. | ce=1.0e-2 | p=0.4e-4) | -> | 609 |
| | 1.0% | T(0.573: | g=1. | ce=8.0e-3 | p=0. ) | -> | 1275 |
| | 1.6% | T(0.470: | g=1. | ce=1.6e-1 | p=0. ) | -> | 1378 |

```
1765   88.5%  T(1.765:  g=1.  ce=4.0e-3  p=1.5e-4)  ->      0
        9.5%  T(1.155:  g=1.  ce=1.1e-2  p=0.3e-4)  ->    609
        2.0%  T(0.387:  g=1.  ce=2.8e-1  p=0.     )  ->   1378

1743          T(1.134:  g=1.  ce=2.2e-3  p=0.1e-4)  ->    609

1730   16.2%  T(1.730:  g=1.  ce=1.7e-3  p=0.8e-4)  ->      0
       82.1%  T(1.120:  g=1.  ce=1.3e-2  p=0.1e-4)  ->    609
        1.7%  T(0.455:  g=1.  ce=1.3e-2  p=0.     )  ->   1275

1661     79%  T(1.661:  g=1.  ce=2.3e-3  p=0.6e-4)  ->      0
         21%  T(1.052:  g=1.  ce=2.0e-2  p=0.     )  ->    609

1543     10%  T(1.543:  g=1.  ce=3.2e-3  p=1.0e-4)  ->      0
         90%  T(0.934:  g=1.  ce=1.9e-2  p=0.     )  ->    609

1415     22%  conv.el.(1.323) + gamma(0.093)        ->      0
         78%  T(0.806:  g=1.  ce=1.1e-2  p=0.     )  ->    609

1378     45%  T(1.378:  g=1.  ce=3.0e-3  p=0.2e-4)  ->      0
         55%  T(0.768:  g=1.  ce=1.2e-2  p=0.     )  ->    609

1275          T(0.665:  g=1.  ce=4.5e-3  p=0.     )  ->    609

 609          T(0.609:  g=1.  ce=2.1e-2  p=0.     )  ->      0
```

Binding energy of K-shell Po electron = 0.093

```
--------------------------------------------------------------------------------
                              Model of Pb214 decay
--------------------------------------------------------------------------------

 100% beta decay to Bi214   Q(beta)=1.024

 2.5%  beta(0.185) -> level 839 keV of Bi214
 1.0%  beta(0.490) ->       534
48.1%  beta(0.672) ->       352
42.1%  beta(0.729) ->       295
 6.3%  beta(1.024) ->         0

level 839  25.9%  T(0.839:  g=1.  ce=4.0e-3  p=0.) -> level   0
           38.8%  T(0.786:  g=1.  ce=4.5e-3  p=0.) ->          53
           16.0%  T(0.580:  g=1.  ce=8.0e-3  p=0.) ->         259
            3.0%  T(0.544:  g=1.  ce=9.0e-3  p=0.) ->         295
           15.0%  T(0.487:  g=1.  ce=1.8e-2  p=0.) ->         352
            1.3%  T(0.305:  g=1.  ce=3.0e-2  p=0.) ->         534

      534    14%  T(0.534:  g=1.  ce=9.0e-3  p=0.) ->           0
             28%  T(0.481:  g=1.  ce=1.8e-2  p=0.) ->          53
             57%  T(0.275:  g=1.  ce=4.0e-2  p=0.) ->         259
              1%  T(0.238:  g=1.  ce=6.5e-2  p=0.) ->         295

      352         T(0.352:  g=1.  ce=2.1e-1  p=0.) ->           0

      295    71%  T(0.295:  g=1.  ce=3.5e-1  p=0.) ->           0
             29%  T(0.242:  g=1.  ce=7.5e-1  p=0.) ->          53

      259    98%  T(0.259:  g=1.  ce=5.8e-1  p=0.) ->           0
              2%  T(0.206:  g=1.  ce=9.2e-1  p=0.) ->          53

       53         T(0.053:  g=1.  ce=9.2e+0  p=0.) ->           0

Binding energy of K-sell Bi electron = 0.091
                  L-shell            = 0.016
```

```
-------------------------------------------------------------------------------
                           Model of Ac228 decay
-------------------------------------------------------------------------------

  100% beta decay to Th228        Q(beta)=2.137

 0.20%   beta(0.110)  -> level 2027 keV of Th228
 0.20%   beta(0.127)  ->       2010
 0.04%   beta(0.172)  ->       1965
 0.23%   beta(0.192)  ->       1945
 0.10%   beta(0.237)  ->       1900
 0.14%   beta(0.244)  ->       1893
 0.11%   beta(0.377)  ->       1760
 0.29%   beta(0.393)  ->       1744
 1.50%   beta(0.413)  ->       1724
 2.10%   beta(0.448)  ->       1689
 1.50%   beta(0.454)  ->       1683
 4.70%   beta(0.491)  ->       1646
 0.80%   beta(0.494)  ->       1643
 1.20%   beta(0.499)  ->       1638
 0.07%   beta(0.590)  ->       1547
 0.20%   beta(0.598)  ->       1539
 8.00%   beta(0.605)  ->       1532
 0.20%   beta(0.648)  ->       1489
 1.90%   beta(0.687)  ->       1450
 1.50%   beta(0.705)  ->       1432
 0.20%   beta(0.793)  ->       1344
 0.80%   beta(0.910)  ->       1227
 3.60%   beta(0.968)  ->       1169
 5.00%   beta(0.983)  ->       1154
 5.40%   beta(1.014)  ->       1123
 0.10%   beta(1.077)  ->       1060
 2.00%   beta(1.115)  ->       1022
 0.20%   beta(1.121)  ->       1016
 0.10%   beta(1.158)  ->        979
31.95%   beta(1.168)  ->        969
 0.30%   beta(1.169)  ->        968
 0.23%   beta(1.193)  ->        944
 0.14%   beta(1.262)  ->        875
 0.20%   beta(1.618)  ->        519
13.00%   beta(1.741)  ->        396
 0.80%   beta(1.950)  ->        187
11.00%   beta(2.079)  ->         58

level 2027    31%  T(1.509:  g=1.  ce=3.2e-3  p=0.      ) -> level  969
              69%  T(1.004:  g=1.  ce=3.0e-3  p=0.      ) ->       1022

      2010    42%  T(1.952:  g=1.  ce=2.8e-3  p=1.6e-4) ->           58
              30%  T(1.823:  g=1.  ce=7.0e-3  p=1.7e-4) ->          187
              28%  T(0.920:  g=1.  ce=0.7e+0  p=0.      ) ->       1091

      1965    33%  T(1.966:  g=1.  ce=1.2e-3  p=2.9e-4) ->            0
              67%  T(1.907:  g=1.  ce=1.8e-3  p=2.7e-4) ->           58

      1945  31.4%  T(1.887:  g=1.  ce=3.0e-3  p=1.3e-4) ->           58
            11.0%  T(1.758:  g=1.  ce=1.0e-2  p=2.0e-4) ->          187
             1.9%  T(1.549:  g=1.  ce=1.7e-3  p=0.6e-4) ->          396
            31.4%  T(0.975:  g=1.  ce=4.0e-2  p=0.      ) ->        969
            12.1%  T(0.922:  g=1.  ce=4.5e-2  p=0.      ) ->       1022
             1.2%  T(0.853:  g=1.  ce=5.5e-2  p=0.      ) ->       1091
             5.0%  T(0.791:  g=1.  ce=6.5e-2  p=0.      ) ->       1154
             2.0%  T(0.745:  g=1.  ce=7.5e-2  p=0.      ) ->       1200
             4.0%  T(0.220:  g=1.  ce=2.1e+0  p=0.      ) ->       1724
```

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1900 | 1.7% | T(1.900: | g=1. | ce=3.0e-3 | p=1.3e-4) | -> | 0 |
| | 19.0% | T(1.842: | g=1. | ce=8.0e-3 | p=2.0e-4) | -> | 58 |
| | 1.0% | T(1.712: | g=1. | ce=3.5e-3 | p=0.8e-4) | -> | 187 |
| | 10.0% | T(1.504: | g=1. | ce=1.7e-3 | p=0.5e-4) | -> | 396 |
| | 36.2% | T(0.884: | g=1. | ce=4.3e-3 | p=0. ) | -> | 1016 |
| | 32.1% | T(0.449: | g=1. | ce=1.6e-2 | p=0. ) | -> | 1450 |
| 1893 | 17.0% | T(1.835: | g=1. | ce=6.0e-3 | p=2.0e-4) | -> | 58 |
| | 5.6% | T(1.706: | g=1. | ce=1.0e-2 | p=1.0e-4) | -> | 187 |
| | 16.0% | T(0.940: | g=1. | ce=1.0e-1 | p=0. ) | -> | 952 |
| | 9.0% | T(0.924: | g=1. | ce=4.5e-2 | p=0. ) | -> | 969 |
| | 27.2% | T(0.870: | g=1. | ce=5.2e-2 | p=0. ) | -> | 1022 |
| | 2.0% | T(0.739: | g=1. | ce=3.3e-1 | p=0. ) | -> | 1154 |
| | 1.0% | T(0.693: | g=1. | ce=1.2e-1 | p=0. ) | -> | 1200 |
| | 22.2% | T(0.461: | g=1. | ce=2.8e-1 | p=0. ) | -> | 1432 |
| 1760 | 62% | T(1.702: | g=1. | ce=1.4e-3 | p=1.1e-4) | -> | 58 |
| | 38% | T(1.574: | g=1. | ce=1.7e-3 | p=0.7e-4) | -> | 187 |
| 1744 | 32% | T(1.686: | g=1. | ce=1.4e-3 | p=1.6e-4) | -> | 58 |
| | 62% | T(1.557: | g=1. | ce=1.7e-3 | p=0.6e-4) | -> | 187 |
| | 6% | T(1.348: | g=1. | ce=2.0e-3 | p=0.2e-4) | -> | 396 |
| 1724 | 2.0% | T(1.724: | g=1. | ce=3.7e-3 | p=0.7e-4) | -> | 0 |
| | 13.0% | T(1.666: | g=1. | ce=1.0e-2 | p=1.2e-4) | -> | 58 |
| | 0.8% | T(1.537: | g=1. | ce=4.0e-3 | p=0.4e-4) | -> | 187 |
| | 73.2% | T(0.755: | g=1. | ce=6.9e-2 | p=0. ) | -> | 969 |
| | 11.0% | T(0.702: | g=1. | ce=9.5e-2 | p=0. ) | -> | 1022 |
| 1689 | 72.3% | T(1.631: | g=1. | ce=7.4e-3 | p=1.2e-4) | -> | 58 |
| | 25.7% | T(1.502: | g=1. | ce=1.7e-3 | p=0.5e-4) | -> | 187 |
| | 2.0% | T(0.666: | g=1. | ce=7.5e-3 | p=0. ) | -> | 1022 |
| 1683 | 22% | T(1.625: | g=1. | ce=1.5e-3 | p=0.8e-4) | -> | 58 |
| | 68% | T(1.496: | g=1. | ce=1.7e-3 | p=0.5e-4) | -> | 187 |
| | 6% | T(1.287: | g=1. | ce=2.2e-3 | p=0.2e-4) | -> | 396 |
| | 4% | T(1.165: | g=1. | ce=2.6e-3 | p=0.1e-4) | -> | 519 |
| 1646 | 75.0% | T(1.588: | g=1. | ce=4.7e-3 | p=0.5e-4) | -> | 58 |
| | 20.0% | T(1.459: | g=1. | ce=5.0e-3 | p=0.3e-4) | -> | 187 |
| | 1.0% | T(0.677: | g=1. | ce=2.2e-2 | p=0. ) | -> | 969 |
| | 0.4% | T(0.624: | g=1. | ce=8.0e-3 | p=0. ) | -> | 1022 |
| | 1.0% | T(0.555: | g=1. | ce=1.6e-1 | p=0. ) | -> | 1091 |
| | 2.1% | T(0.523: | g=1. | ce=1.2e-2 | p=0. ) | -> | 1123 |
| | 0.5% | T(0.420: | g=1. | ce=1.8e-2 | p=0. ) | -> | 1227 |
| 1643 | 1.0% | T(1.315: | g=1. | ce=1.8e-2 | p=0.6e-4) | -> | 328 |
| | 50.7% | T(1.247: | g=1. | ce=2.1e-2 | p=0.4e-4) | -> | 396 |
| | 10.0% | T(0.675: | g=1. | ce=7.0e-3 | p=0. ) | -> | 969 |
| | 9.0% | T(0.620: | g=1. | ce=8.0e-3 | p=0. ) | -> | 1022 |
| | 29.3% | T(0.210: | g=1. | ce=7.9e-2 | p=0. ) | -> | 1432 |
| 1638 | 38% | T(1.638: | g=1. | ce=4.0e-3 | p=0.5e-4) | -> | 0 |
| | 58% | T(1.581: | g=1. | ce=1.1e-2 | p=1.1e-4) | -> | 58 |
| | 4% | T(0.516: | g=1. | ce=1.2e-2 | p=0. ) | -> | 1123 |
| 1547 | 50% | T(1.169: | g=1. | ce=2.7e-3 | p=0.2e-4) | -> | 378 |
| | 36% | T(0.378: | g=1. | ce=2.2e-2 | p=0. ) | -> | 1169 |
| | 14% | T(0.373: | g=1. | ce=2.2e-2 | p=0. ) | -> | 1175 |
| 1539 | 12% | T(1.480: | g=1. | ce=1.8e-3 | p=0.5e-4) | -> | 58 |
| | 6% | T(1.143: | g=1. | ce=2.6e-2 | p=0.3e-4) | -> | 396 |
| | 14% | T(1.020: | g=1. | ce=1.0e-2 | p=0. ) | -> | 519 |
| | 68% | T(0.571: | g=1. | ce=1.5e-1 | p=0. ) | -> | 968 |
| 1532 | 0.4% | T(1.136: | g=1. | ce=2.8e-3 | p=0.2e-4) | -> | 396 |

```
        30.3%  T(0.563:  g=1.  ce=5.0e-2  p=0.      ) ->        969
        16.0%  T(0.509:  g=1.  ce=6.0e-2  p=0.      ) ->       1022
         4.6%  T(0.441:  g=1.  ce=3.0e-1  p=0.      ) ->       1091
         1.0%  T(0.378:  g=1.  ce=4.5e-1  p=0.      ) ->       1154
         0.7%  T(0.357:  g=1.  ce=1.7e+0  p=0.      ) ->       1175
        47.0%  T(0.100:  g=1.  ce=4.0e+0  p=0.      ) ->       1432

1489      22%  T(0.399:  g=1.  ce=2.0e-2  p=0.      ) ->       1091
          78%  T(0.314:  g=1.  ce=0.6e+0  p=0.      ) ->       1175

1450     3.0%  T(1.054:  g=1.  ce=3.2e-2  p=0.      ) ->        396
        12.0%  T(0.498:  g=1.  ce=4.2e-2  p=0.      ) ->        952
         1.5%  T(0.481:  g=1.  ce=2.5e-1  p=0.      ) ->        968
        41.5%  T(0.328:  g=1.  ce=4.4e-2  p=0.      ) ->       1123
        24.0%  T(0.282:  g=1.  ce=1.3e+0  p=0.      ) ->       1169
        18.0%  T(0.224:  g=1.  ce=1.5e+0  p=0.      ) ->       1227

1432     1.0%  T(1.374:  g=1.  ce=1.4e-2  p=0.7e-4) ->         58
         2.0%  T(1.245:  g=1.  ce=2.0e-2  p=0.4e-4) ->        187
        61.8%  T(0.463:  g=1.  ce=4.7e-2  p=0.      ) ->        969
        27.4%  T(0.410:  g=1.  ce=8.3e-2  p=0.      ) ->       1022
         6.0%  T(0.341:  g=1.  ce=1.2e-1  p=0.      ) ->       1091
         0.8%  T(0.308:  g=1.  ce=3.5e-2  p=0.      ) ->       1123
         0.6%  T(0.264:  g=1.  ce=5.0e-2  p=0.      ) ->       1169
         0.4%  T(0.258:  g=1.  ce=5.0e-2  p=0.      ) ->       1175

1344    27.0%  T(1.017:  g=1.  ce=3.5e-3  p=0.      ) ->        328
        37.5%  T(0.948:  g=1.  ce=3.7e-3  p=0.      ) ->        396
        30.5%  T(0.825:  g=1.  ce=5.0e-3  p=0.      ) ->        519
         5.0%  T(0.169:  g=1.  ce=1.4e-1  p=0.      ) ->       1175

1227      10%  T(1.040:  g=1.  ce=3.5e-3  p=0.      ) ->        187
          58%  T(0.830:  g=1.  ce=1.8e-2  p=0.      ) ->        396
          11%  T(0.707:  g=1.  ce=1.0e-1  p=0.      ) ->        519
          18%  T(0.204:  g=1.  ce=9.0e-2  p=0.      ) ->       1022
           3%  T(0.136:  g=1.  ce=1.7e+0  p=0.      ) ->       1091

1200           T(0.178:  g=1.  ce=6.0e+1  p=0.      ) ->       1022

1175      67%  T(0.988:  g=1.  ce=3.5e-3  p=0.      ) ->        187
          33%  T(0.796:  g=1.  ce=5.2e-3  p=0.      ) ->        378

1169    11.0%  T(1.110:  g=1.  ce=2.9e-3  p=0.1e-4) ->         58
        27.8%  T(0.840:  g=1.  ce=1.4e-2  p=0.      ) ->        328
        45.7%  T(0.772:  g=1.  ce=1.5e-2  p=0.      ) ->        396
         0.3%  T(0.649:  g=1.  ce=2.4e-2  p=0.      ) ->        519
         9.0%  T(0.200:  g=1.  ce=9.5e-2  p=0.      ) ->        969
         5.1%  T(0.146:  g=1.  ce=1.2e+0  p=0.      ) ->       1022
         1.1%  T(0.078:  g=1.  ce=2.2e-1  p=0.      ) ->       1091

1154    14.9%  T(1.154:  g=1.  ce=7.5e-3  p=0.1e-4) ->          0
        10.9%  T(1.096:  g=1.  ce=2.8e-2  p=0.1e-4) ->         58
        14.9%  T(0.967:  g=1.  ce=2.0e-2  p=0.      ) ->        187
        20.7%  T(0.322:  g=1.  ce=5.2e-1  p=0.      ) ->        832
        22.7%  T(0.279:  g=1.  ce=1.3e+0  p=0.      ) ->        875
         8.9%  T(0.185:  g=1.  ce=5.4e+1  p=0.      ) ->        969
         4.0%  T(0.174:  g=1.  ce=1.4e+0  p=0.      ) ->        979
         3.0%  T(0.138:  g=1.  ce=4.9e+0  p=0.      ) ->       1016

1123     1.4%  T(1.123:  g=1.  ce=6.5e-2  p=0.1e-4) ->          0
         4.0%  T(1.065:  g=1.  ce=3.2e-3  p=0.      ) ->         58
         0.2%  T(0.936:  g=1.  ce=0.1e+0  p=0.      ) ->        187
        68.2%  T(0.795:  g=1.  ce=1.9e-2  p=0.      ) ->        328
        12.0%  T(0.727:  g=1.  ce=1.0e-2  p=0.      ) ->        396
        12.3%  T(0.154:  g=1.  ce=0.2e+0  p=0.      ) ->        969
         1.9%  T(0.100:  g=1.  ce=3.3e-1  p=0.      ) ->       1022
```

```
1091    21%  T(1.033:  g=1.  ce=9.5e-3  p=0.    ) ->        58
        79%  T(0.904:  g=1.  ce=2.6e-2  p=0.    ) ->       187

1060    74%  T(1.002:  g=1.  ce=3.5e-3  p=0.    ) ->        58
        13%  T(0.664:  g=1.  ce=7.0e-3  p=0.    ) ->       396
        13%  T(0.541:  g=1.  ce=1.1e-2  p=0.    ) ->       519

1022    73%  T(0.964:  g=1.  ce=9.2e-3  p=0.    ) ->        58
        27%  T(0.836:  g=1.  ce=1.4e-2  p=0.    ) ->       187

1016    19%  T(1.017:  g=1.  ce=2.4e-2  p=0.    ) ->         0
        47%  T(0.958:  g=1.  ce=3.8e-3  p=0.    ) ->        58
        16%  T(0.688:  g=1.  ce=2.1e-2  p=0.    ) ->       328
        18%  T(0.620:  g=1.  ce=1.4e-1  p=0.    ) ->       396

 979    35%  T(0.651:  g=1.  ce=[illegible]5e-3  p=0.    ) ->       328
        65%  T(0.583:  g=1.  ce=[illegible].5e-3  p=0.    ) ->       396

 969  37.0%  T(0.969:  g=1.  ce=1.0e-2  p=0.    ) ->         0
      61.8%  T(0.911:  g=1.  ce=1.2e-2  p=0.    ) ->        58
       1.2%  T(0.782:  g=1.  ce=6.8e-2  p=0.    ) ->       187

 968    22%  T(0.641:  g=1.  ce=1.1e-1  p=0.    ) ->       328
        78%  T(0.572:  g=1.  ce=1.3e-1  p=0.    ) ->       396

 952    91%  T(0.894:  g=1.  ce=4.2e-3  p=0.    ) ->        58
         3%  T(0.624:  g=1.  ce=1.2e-1  p=0.    ) ->       328
         6%  T(0.556:  g=1.  ce=3.5e-2  p=0.    ) ->       396

 944  42.5%  T(0.944:  g=1.  ce=1.1e-2  p=0.    ) ->         0
      15.0%  T(0.888:  g=1.  ce=7.5e-1  p=0.    ) ->        58
      42.5%  T(0.616:  g=1.  ce=8.5e-3  p=0.    ) ->       328

 875    15%  T(0.874:  g=1.  ce=1.3e-2  p=0.    ) ->         0
        11%  T(0.816:  g=1.  ce=0.5e+0  p=0.    ) ->        58
        34%  T(0.546:  g=1.  ce=1.1e-2  p=0.    ) ->       328
        40%  T(0.479:  g=1.  ce=1.4e-2  p=0.    ) ->       396

 832     4%  conv.el.(0.721) + gamma(0.110)       ->         0
        23%  T(0.771:  g=1.  ce=1.7e-2  p=0.    ) ->        58
        73%  T(0.504:  g=1.  ce=1.2e-2  p=0.    ) ->       328

 519    90%  T(0.332:  g=1.  ce=4.7e-1  p=0.    ) ->       187
        10%  T(0.141:  g=1.  ce=0.9e+0  p=0.    ) ->       378

 396    73%  T(0.338:  g=1.  ce=1.0e-2  p=0.    ) ->        58
        27%  T(0.209:  g=1.  ce=7.9e-2  p=0.    ) ->       187

 378         T(0.191:  g=1.  ce=4.2e-1  p=0.    ) ->       187

 328    47%  T(0.328:  g=1.  ce=4.4e-2  p=0.    ) ->         0
        53%  T(0.270:  g=1.  ce=3.4e-2  p=0.    ) ->        58

 187         T(0.129:  g=1.  ce=2.7e+0  p=0.    ) ->        58

  58         T(0.058:  g=1.  ce=1.2e+2  p=0.    ) ->         0

Binding energy of K-shell Th electron = 0.110
                  L-shell             = 0.020
```

```
--------------------------------------------------------------------------------
                          Model of Pa234m decay
--------------------------------------------------------------------------------

Approximations:
1) isomeric transition Pa234m to Pa234 is not considered (p=0.13%
   and scheme of Pa234 further decay to U234 is not very well
   determined);
2) decays of Pa234m to excited levels of U234 with energies
   greater than 1.045 MeV are not considered (p=0.17%).

  100% beta decay to U234      Q(beta)=2.287

 0.70%  beta(1.242) -> level 1045 keV of U234
 0.60%  beta(1.477) ->        810
 0.02%  beta(1.501) ->        786
98.68%  beta(2.287) ->          0

level 1045   80.13%  T(1.001:  g=1.  ce=1.0e-2  p=0.) -> level  43
              7.80%  T(0.258:  g=1.  ce=5.0e-2  p=0.) ->       786
             12.00%  conv.el.(0.119) + gamma(0.116)    ->       810
              0.07%  T(0.193:  g=1.  ce=7.0e-1  p=0.) ->       852

       852      56%  T(0.852:  g=1.  ce=1.4e-2  p=0.) ->         0
                33%  T(0.808:  g=1.  ce=2.0e-2  p=0.) ->        43
                11%  T(0.708:  g=1.  ce=2.0e-2  p=0.) ->       143

       810      67%  conv.el.(0.694) + gamma(0.116)    ->         0
                33%  T(0.766:  g=1.  ce=1.5e-2  p=0.) ->        43

       786      38%  T(0.786:  g=1.  ce=5.5e-3  p=0.) ->         0
                62%  T(0.743:  g=1.  ce=6.0e-3  p=0.) ->        43

       143           T(0.100:  g=1.  ce=1.2e+1  p=0.) ->        43

        43           conv.el.(0.021) + gamma(0.022)    ->         0

Binding energy of K-shell U electron = 0.116
                  L-shell              0.022
```

---

## 4. Check-up on the model's subroutines

---

Two ideas proved to be very useful to write the subroutines for decay models:

(1) small independent subroutine nucltrans to sample what kind of radiation (gamma or conversion electron or pair) will be emitted in nuclear transition;

(2) the use of labels equal to energies of excited levels of daughter nucleus.

In such approach subroutines for decay models became a simple copy of nuclear schemes [2], their text is very readable and can easily be checked.

Only one subroutine - for 208-Tl - was checked in additional way: in straight manner when probabilities to occupy different excited levels of 208-Pb and to emit gamma quanta with different energies were obtained after 10 millions calls of subroutine. Results are shown in table 2; one can see the full agreement between simulated and theoretical values.

Table 2. Results of test for subroutine Tl208 (10 million events)

1. Probabilities (%) to occupy different excited levels of 208-Pb

| energy of level (keV) | simulated value | theoretical value [2] |
|---|---|---|
| 4481 | 5.0260000e-02 | 5.2e-2 |
| 4383 | 1.6430000e-02 | 1.7e-2 |
| 4358 | 4.3060001e-02 | 4.3e-2 |
| 4296 | 8.1409998e-02 | 8.2e-2 |
| 4180 | 0.2180500 | 0.22 |
| 4125 | 0.1602600 | 0.16 |
| 3996 | 6.9700000e-03 | 7.0e-3 |
| 3961 | 3.093130 | 3.1 |
| 3946 | 4.0089998e-02 | 4.0e-2 |
| 3920 | 0.5698900 | 0.57 |
| 3709 | 22.81095 | 22.8 |
| 3475 | 21.71441 | 21.7 |
| 3198 | 51.16584 | 51.2 |
| 2615 | 2.9250000e-02 | 3.0e-2 |

2. Probabilities (%) to emit different gamma quanta

| energy of gamma (keV) | simulated value | theoretical value [2] |
|---|---|---|
| 1283 | 5.0260000e-02 | 5.2e-2 |
| 1185 | 1.6430000e-02 | 1.7e-2 |
| 1744 | 2.1400000e-03 | 2.1e-3 |
| 1161 | 1.0730000e-02 | 1.1e-2 |
| 883 | 3.0190000e-02 | 3.0e-2 |
| 821 | 4.0819999e-02 | 4.1e-2 |
| 589 | 4.0589999e-02 | 4.1e-2 |
| 983 | 0.1961800 | 0.198 |
| 705 | 2.1870000e-02 | 2.2e-2 |
| 928 | 0.1248700 | 0.125 |
| 650 | 3.5390001e-02 | 3.5e-2 |
| 1381 | 6.9700000e-03 | 7.0e-3 |
| 763 | 2.036440 | 2.05 |
| 486 | 6.2689997e-02 | 6.2e-2 |
| 253 | 0.9940000 | 0.99 |
| 749 | 4.0089998e-02 | 4.0e-2 |
| 722 | 0.3061900 | 0.308 |
| 211 | 0.2637000 | 0.262 |
| 1094 | 0.4107500 | 0.41 |
| 511 | 23.36145 | 23.3 |
| 233 | 0.3370400 | 0.337 |
| 860 | 14.23362 | 14.2 |
| 277 | 8.008790 | 8.0 |
| 583 | 85.31727 | 85.3 |
| 2615 | 100.0000 | 100.0 |

## 5. Some notes about previous models for nuclide decay

Models of decay were constructed previously for a few nuclides: 208-Tl, 212-Bi, 214-Bi and 214-Pb. May be it will be useful to make some comparison between old and new models.

(1) Old models were approximations to "real" nuclear schemes of [2] and took into account the decay branches with big probabilities only. As a result of simplifications and artificial unification of different branches gamma quanta with nonexisting energies, or beta particles with nonexisting Qb, or nonexisting combinations of transitions appeared sometimes. For example,
  - model of 208-Tl accounted for beta decay to 8 excited levels of daughter nucleus and for 15 different nuclear transitions (4 of them had nonstandard energies differed from values of [2] up to a few tens of keV); really 14 levels and 25 transitions are listed in [2];
  - model of 212-Bi accounted for 9 excited levels (2 of them were nonstandard) and 7 transitions (2 nonstandard); [2] gives us the list of 7 levels and 11 transitions;
  - model of 214-Bi considered 18 levels (7 nonstandard) and 20 transitions; in [2] we have 48 levels and 103 transitions (for beta decay);
  - model of 214-Pb - 3 levels and 5 transitions; in [2] we have 5 levels and 16 transitions;

(2) energy was conserved approximately - in different branches of decay of the same nuclide energy release fluctuated:
  - for 208-Tl - from 4.889 to 5.017 MeV instead of 4.992 [2]; in one branch emission of gamma 2.615 MeV was missed (but corresponding probability is low - ~0.4%);
  - for 212-Bi - from 2.127 to 2.273 instead of 2.246 MeV [2];
  - for 214-Bi - from 3.200 to 3.297 instead of 3.270 MeV [2];

(3) values of electron conversion coefficient were taken in old models from another source than [2] or [6] apparently. There are quite big differences between old and new coefficients sometimes (for example 5.7 instead of 0.75 (experimental value [2]) for transition 0.242 MeV in 214-Pb decay);

(4) some very interesting from the point of view of imitating double beta decay branches were omitted (for example emission of beta particle with Qb=1.855 MeV and conversion electron 1.323 MeV with probability 0.22% in 214-Bi decay);

(5) emission of only one conversion electron in cascade of transitions was accounted previously. New models permit emission of two or more conversion electrons;

(6) possibility to emit electron-positron pair instead of gamma was not taken into consideration at all;

(7) energy spectrum of beta particles was displaced to lower energies (see section 6).

Nevertheless, old models described the nuclides decay correctly in the main.

---

## 6. Improvement on sampling the kinetic energy of electrons emitted in beta decay

---

If we confine ourselves to consider only allowed beta decays (1) and only the Coulomb field corrections (2) the energy spectrum of particles emitted in beta decay can be described as

$$N(Eb)=Pb*Pn*Wb*Wn*F(Z,Eb), \quad (1)$$

where E/W - kinetic/full energy of particle (b - beta particle, n - neutrino),
P - particle's momentum,
Z - atomic number of daughter nucleus,
F(Z,Eb) - factor Fermi accounting for the nuclear Coulomb potential.
Because of Pb=sqrt(Eb*(Eb+1.022 MeV)),
Wb=Eb+0.511 MeV,
Pn=Wn=Qb-Eb (Qb - energy release in beta decay),
another form of (1) is

$$N(Eb)=sqrt(Eb*(Eb+1.022))*(Eb+0.511)*(Qb-Eb)**2*F(Z,Eb). \quad (2)$$

Factor Fermi

$$F(Z,Eb)=const*Pb**(2*g-2)*exp(pi*y)*[abs(G(g+i*y))]**2, \quad (3)$$

where g = sqrt(1-(alpha*Z)**2),
alpha = 1/137.036,
pi = 3.14159...,
y = alpha*Z*Wb/Pb,
G - gamma function of complex argument.

In old subroutine beta for simulation of beta particle energy following expression for spectrum was used:

$$N(Eb)=Pn*Wb*Wn. \quad (4)$$

So, factor Fermi was not accounted but also momentum Pb was missed.

Differences between spectrum shapes are shown in fig. 4 for isotopes for which models were constructed previously - 208-Tl, 212-Bi, 214-Bi and 214-Pb. Curve 1 on all pictures is calculated by "old" expression PnWbWn, curve 2 - by PbPnWbWn (correct kinematical factor without Coulomb correction), curve 3 - by PbPnWbWnF. Beta decay can occur with different Qb for these isotopes, and values of Qb and corresponding probabilities are shown in pictures. All curves are normalised for area = 1. Factors Pb and F compensate each other for these nuclides to some extent but not completely of course. As a result, accounting of both factors PbF displaces the spectrum of beta particles to higher energies.

New subroutine beta1 was written to simulate the energy of beta particles in accordance with expression (1). It allows to consider not only electrons but also positrons emitted in beta decay (Z>0 for e- and Z<0 for e+ in F(Z,Eb) factor). Subroutine cgamma of CERNlibrary was used to calculate the gamma function of complex argument. Because of more complicated expression new subroutine beta1 demands near 10 minutes additionally (on VAX 6520) as compared with old subroutine beta to simulate 100 thousand of beta events; this time looks as acceptable. Results of simulation of 10 thousand events are shown in fig. 5 for beta decay with energy release 2.5 MeV but different Z of daughter nucleus : a - Z=0 (no Coulomb field), b - Z=+90 (beta- decay), c - Z=-90 (beta+ decay); one can see the essential displacement of energy spectrum in result of nuclear field influence.

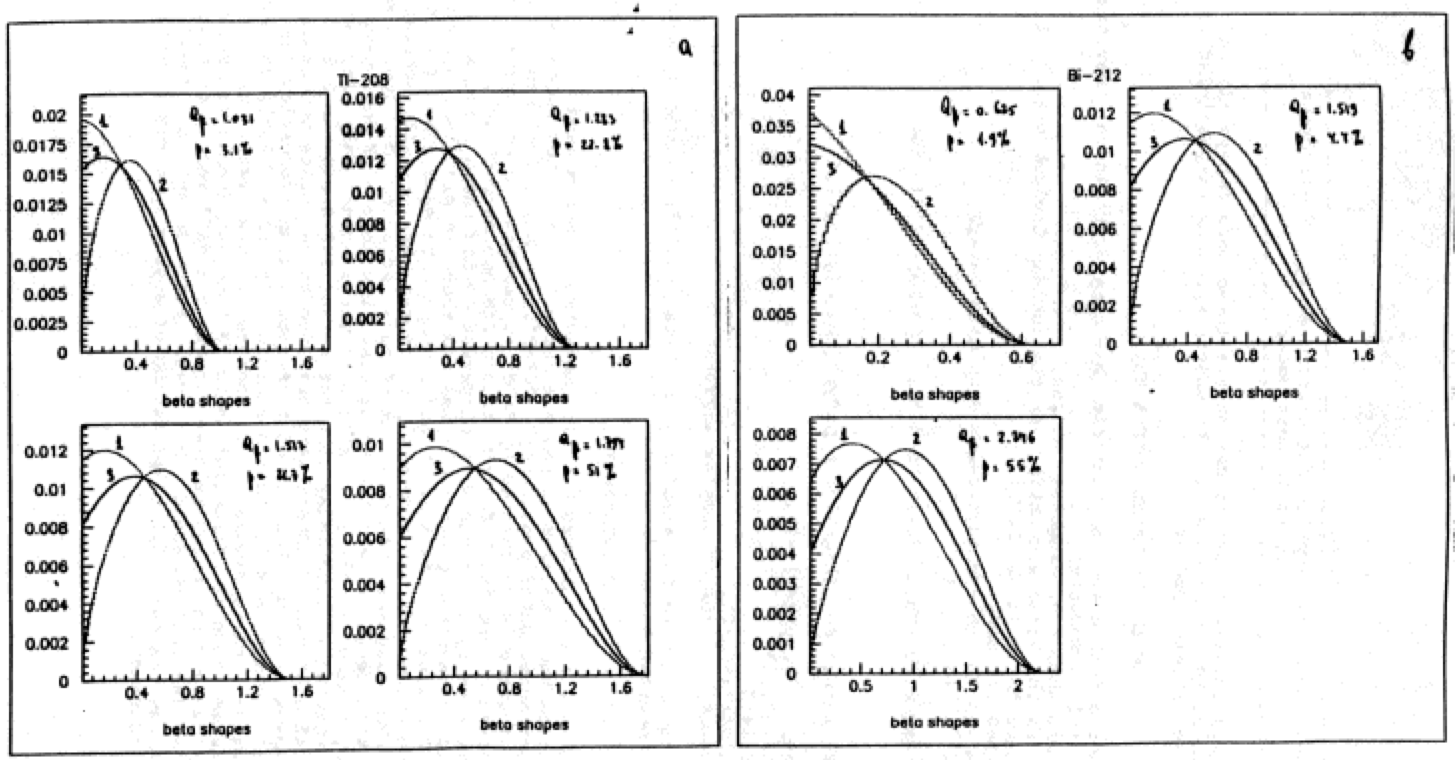



**Fig. 4a,4b. Shapes of beta spectrum: a - for 208-Tl,
b - for 212-Bi.**

**Curve 1 - expression PnWbWn,
2 - PbPnWbWn,
3 - PbPnWbWnF.**

**Values of Qb and probabilities for different branches of beta decay are shown in figures. The x axis - energy in MeV**

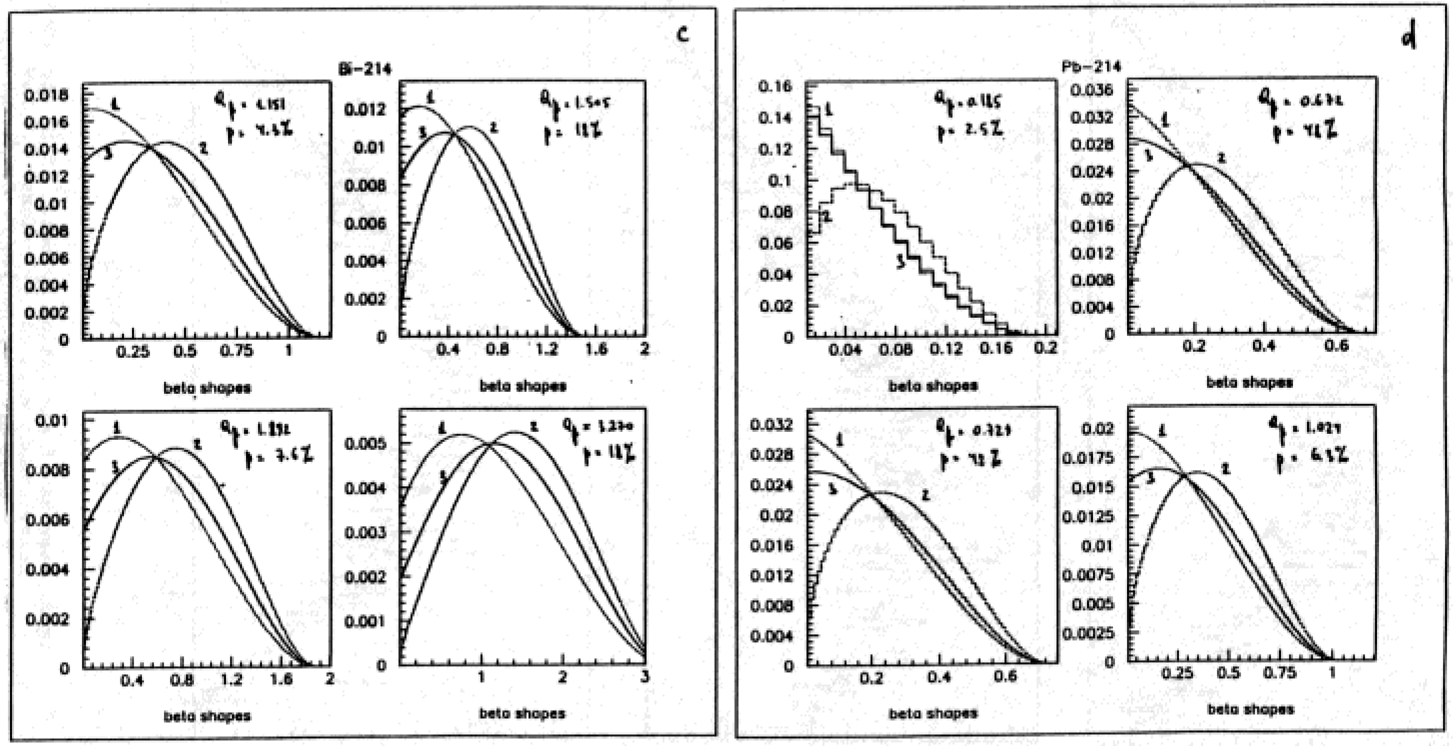



Fig. 4c,4d. Shapes of beta spectrum: c - for 214-Bi,
d - for 214-Pb.

Curve 1 - expression PnWbWn,
2 - PbPnWbWn,
3 - PbPnWbWnF.

Values of Qb and probabilities for different branches of beta decay are shown in figures. The x axis - energy in MeV

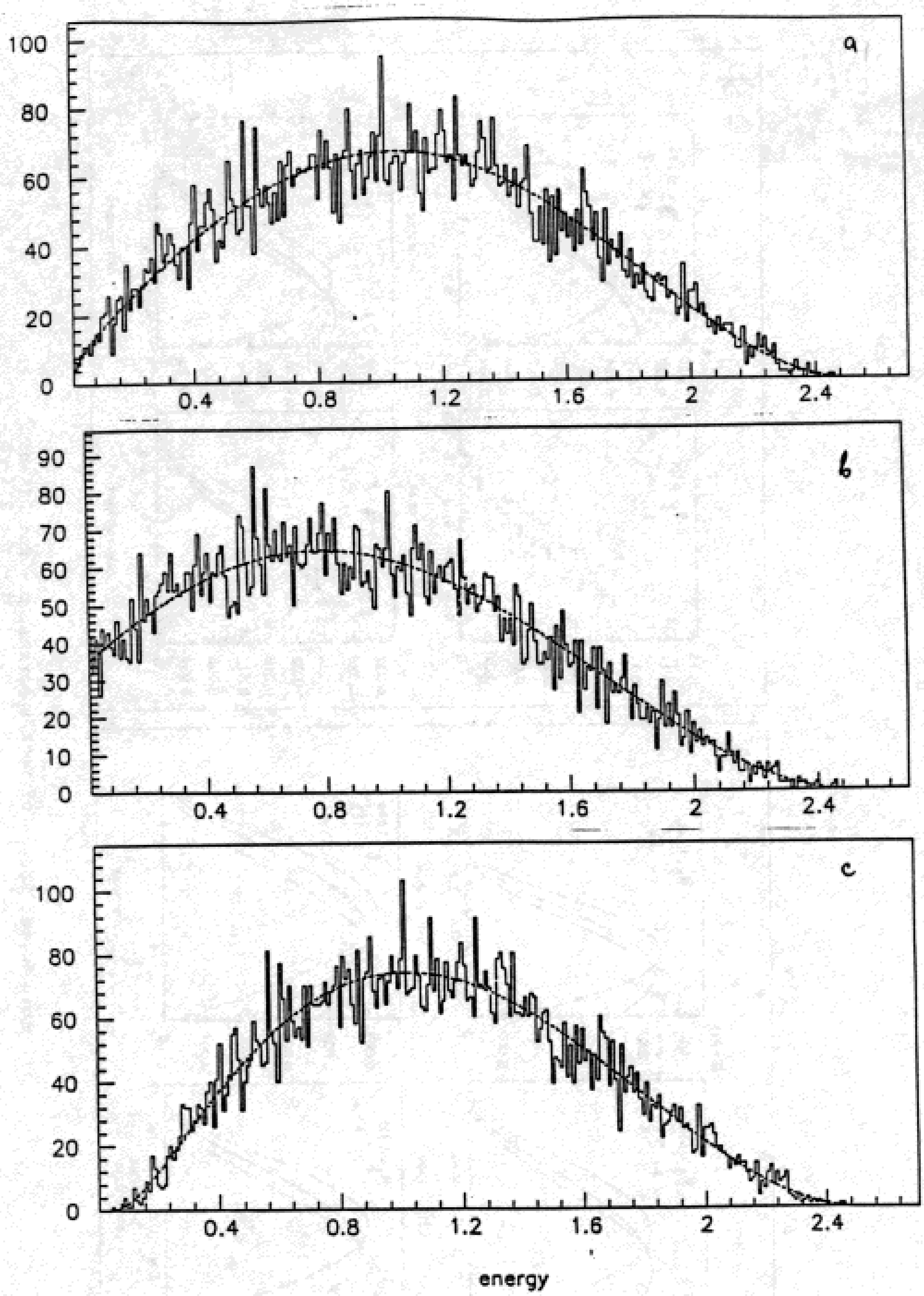



**Fig.5. Results of simulation by beta1 of 10 thousand beta decay events with energy release 2.5 MeV:**
**a - Z of daughter nucleus = 0, no Coulomb field;**
**b - Z=+90 (beta- decay);**
**c - Z=-90 (beta+ decay).**
**Smooth curves are corresponding theoretical distributions**

## 7. Current subroutine for simulation of pair creation

Energy distribution and angular correlation between electron and positron in emitted in deexcitation process pair depend not only on transition energy and atomic number of nucleus but also on type of transition (electrical or magnetic) and its multipolarity. Examples of theoretical expressions one can find in [8] but quantitatively this process can be described in following way:

(1) if one don't consider the effects of Coulomb field, energy distribution for e+ is the same as for e-, distribution has a big width and is symmetrical about point (E(transition)-1.022 MeV)/2. Influence of electrical field of nucleus on emitted charged particles displaces the spectrum of e+ to larger and e- - to less energies;

(2) angle between e+ and e- is small mainly; with increase of Z angular distribution become more isotropical.

The examples of energy and angular distributions are shown in fig. 6.

Current subroutine for simulation of pair creation is only a zero-approximation to real description of this process. It assumes that:

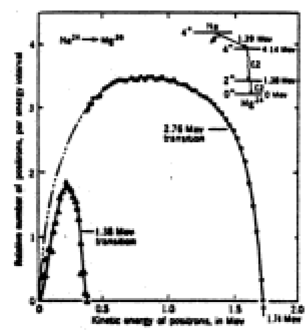
(1) e+ and e- have equal energies;

(2) they are emitted in the same direction.

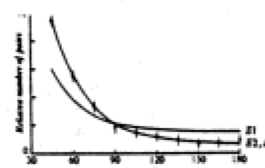
Fig. 6 shows that these assumptions have physical sense although quite rough. We must keep in mind also that:

(1) probability of pair creation is low (up to 8.5e-4 for all transitions listed in section 3);

(2) type and multipolarity of transitions are unknown often; so, one is uncertain about what theoretical expressions have to be used for energy and angular distributions.




Energy spectra of the positrons produced by pair internal conversion, in comparison with the 2.76-Mev and 1.38-Mev γ-ray transitions which follow the β decay of $Na^{24}$. The area under these curves, when compared with the area under the associated negatron spectrum of β rays and pair conversion negatrons, gives for the pair internal-conversion coefficients 7.1 × $10^{-4}$ for the 2.76-Mev transition and 0.6 × $10^{-4}$ for the 1.38-Mev transition. [From S. D. Bloom (B80).] The predominant mode of decay of $Na^{24}$ is shown in the inset. Both the γ transitions are E2 (based on K-shell internal-conversion coefficients, pair internal-conversion coefficients, and γ-γ angular correlation); the β transition is allowed (log ft = 6.11). A competing feeble (0.003 per cent) 4.17-Mev, second-forbidden β transition is omitted in the figure, as is also a 0.04-per cent crossover E4 γ-ray transition.




Angular correlation between positron and electron for the high energy transition in the $^7Li(p,\gamma)^8Be$ reaction. The full lines are theoretical curves (Devons and Goldring, Proc. phys. Soc. Lond., 67, 413, 1954).

**Fig.6. Energy distribution of e+ and angular correlation between e+ and e- in emitted pair**

CONCLUSION

(1) Models of decay are created for all 13 nuclides what are considered currently as dangerous from the point of view to produce background events to double beta decay of 100-Mo;

(2) decay models are full and practically exact copy of nuclear schemes [2] taking into account up to 48 excited levels of daughter nuclei and up to 166 different transitions between levels in deexcitation process;

(3) possibilities of all concurent processes - to emit gamma quantum or conversion electron or electron-positron pair - are taken into account for all transitions;

(4) improvements are introduced into the way of simulation of beta particles energy spectrum accounting for influence of nuclear Coulomb field on emitted electron or positron.

REFERENCES

1. Ph.Hubert, "Radiopurity of molybdenum", NEMO Collaboration Meeting, Kiev-Ukraine, 12-17.01.1992, pp. 217-223
2. "Table of isotopes", ed. by C.M.Lederer and V.S.Shirley, 7-th ed., 1978
3. F.Laplanche, private communication
4. H.Behrens, W.Buhring, "Electron radial wave functions and nuclear beta decay", Oxford, 1982
5. D.H.Wilkinson, "Evaluation of beta-decay. I. The traditional phase space factors", Nucl. Instrum. Meth. A275(1989)378-386
   D.H.Wilkinson, "Evaluation of beta-decay. II. Finite mass and size effects", Nucl. Instrum. Meth. A290(1990)509-515
6. E.Browne, R.B.Firestone, "Table of radioactive isotopes", ed. by V.S.Shirley, 1986
7. M.Deutch, O.Kofoed-Hansen, "Gamma-Rays", in "Experimental nuclear physics", ed. by E.Segre, v.3, 1959
   Graphs on page 368 reproduced the data of:
   M.E.Rose, Phys. Rev. 76(1949)678
   J.C.Jaeger, H.R.Hulme, Proc. Roy. Soc. (London) 148A(1935)708
8. R.Wilson, "Internal pair creation", in "Experimental nuclear physics", ed. by E.Segre, v.3, 1959