

Precision Measurement of the ${}^7\text{Be}$ Solar Neutrino Interaction Rate in Borexino

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The rate of neutrino-electron elastic scattering interactions from 862 keV ${}^7\text{Be}$ solar neutrinos in Borexino is determined to be $46.0 \pm 1.5(\text{stat})_{-1.6}^{+1.5}(\text{syst})$ counts/(day \cdot 100 ton). This corresponds to a ν_e -equivalent ${}^7\text{Be}$ solar neutrino flux of $(3.10 \pm 0.15) \times 10^9 \text{ cm}^{-2} \text{ s}^{-1}$ and, under the assumption of ν_e transition to other active neutrino flavours, yields an electron neutrino survival probability of 0.51 ± 0.07 at 862 keV. The no flavor change hypothesis is ruled out at 5.0σ . A global solar neutrino analysis with free fluxes determines $\Phi_{pp} = 6.06_{-0.06}^{+0.02} \times 10^{10} \text{ cm}^{-2} \text{ s}^{-1}$ and $\Phi_{\text{CNO}} < 1.3 \times 10^9 \text{ cm}^{-2} \text{ s}^{-1}$ (95% C.L.). These results significantly improve the precision with which the Mikheyev-Smirnov-Wolfenstein large mixing angle neutrino oscillation model is experimentally tested at low energy.

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In the past 40 years, solar neutrino experiments [1–4] have revealed important information about the Sun [5] and have shown that solar neutrinos undergo flavor transitions that are well described by Mikheyev-Smirnov-Wolfenstein large mixing angle (“MSW-LMA”) type flavor oscillations [6]. Reactor antineutrino measurements [7] also support this model. The MSW model predicts a transition in the solar ν_e survival probability (“ P_{ee} ”) at neutrino energies

of about 1–4 MeV. This transition is currently poorly tested. Therefore, in order to test MSW-LMA more thoroughly, to probe other proposed neutrino oscillation scenarios [8], and to further improve our understanding of the Sun, it is important that experimental measurements of the low energy solar neutrino fluxes be improved [9]. At 862 keV, the abundant, monoenergetic ${}^7\text{Be}$ solar neutrinos can provide a precise probe of the survival probability in

this interesting region. In addition, a precise determination of the ${}^7\text{Be}$ flux combined with existing results from radiochemical experiments [1,2] yields improved constraints on the pp and CNO solar neutrino fluxes.

The Borexino experiment at Gran Sasso [10] detects neutrinos through the neutrino-electron elastic scattering interaction on a ~ 278 metric ton liquid scintillator target. The low energy backgrounds in the detector have been suppressed to unprecedented levels [11], making Borexino the first experiment capable of making spectrally resolved measurements of solar neutrinos at energies below 1 MeV. We have previously reported a direct measurement of the ${}^7\text{Be}$ solar neutrino flux with combined statistical and systematic errors of 10% [12]. Following a campaign of detector calibrations and a fourfold increase in solar neutrino exposure, we present here a new ${}^7\text{Be}$ neutrino flux measurement with a total uncertainty less than 5%. For the first time, the experimental uncertainty is smaller than the uncertainty in the Standard Solar Model (“SSM”) prediction of the ${}^7\text{Be}$ neutrino flux [13,14].

The new result is based on the analysis of 740.7 live-days (after cuts) of data which were recorded in the period from May 16, 2007 to May 8, 2010, and which correspond to a 153.6 ton \cdot yr fiducial exposure.

The experimental signature of ${}^7\text{Be}$ neutrino interactions in Borexino is a Compton-like shoulder at ~ 660 keV. Fits to the spectrum of observed event energies are used to distinguish between this neutrino scattering feature and backgrounds from radioactive decays [12]. Two independent fit methods were used, one which is Monte Carlo based and one which uses an analytic description of the detector response. In both methods, the weights for the ${}^7\text{Be}$ neutrino signal and the main radioactive background components (${}^{85}\text{Kr}$, ${}^{210}\text{Po}$, ${}^{210}\text{Bi}$, and ${}^{11}\text{C}$) were left as free parameters in the fit, while the contributions of the pp , pep , CNO, and ${}^8\text{B}$ solar neutrinos were fixed to the SSM-predicted rates assuming MSW neutrino oscillations with $\tan^2\theta_{12} = 0.47_{-0.04}^{+0.05}$ and $\Delta m_{12}^2 = (7.6 \pm 0.2) \times 10^{-5} \text{ eV}^2$ [15]. The impact of fixing these fluxes was evaluated and included as a systematic uncertainty. The rates of ${}^{222}\text{Rn}$, ${}^{218}\text{Po}$, and ${}^{214}\text{Pb}$ surviving the cuts were fixed using the measured rate of ${}^{214}\text{Bi}$ - ${}^{214}\text{Po}$ delayed coincidence events. The Monte Carlo method also includes external γ -ray background, which makes it possible to extend the fit range in this method to higher energies. The energy scale and resolution were floated in the analytic fits, while the Monte Carlo approach automatically incorporates the simulated energy response of the detector.

The stability of each fit method was studied by repeating the fits with slightly varied fit characteristics (e.g., fit range and histogram binning) and different methods of data preparation. The latter included changing the method used to estimate the event energies, and varying the pulse shape analysis (“PSA”) technique [16] used to remove ${}^{210}\text{Po}$ and other α events between a highly efficient

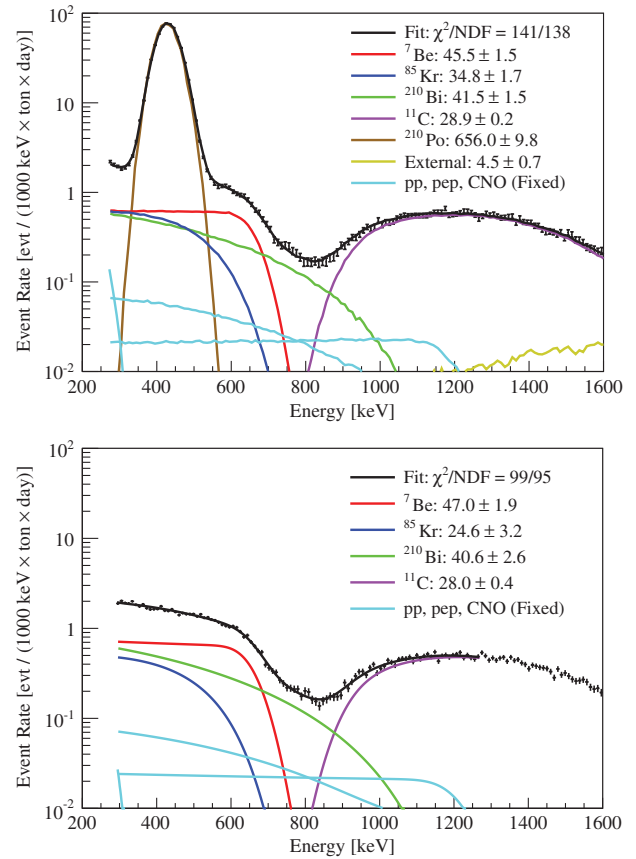


FIG. 1 (color). Two example fitted spectra; the fit results in the legends have units [counts/(day \cdot 100 ton)]. Top: A Monte Carlo based fit over the energy region 270–1600 keV to a spectrum from which some, but not all, of the α events have been removed using a PSA cut, and in which the event energies were estimated using the number of photons detected by the PMT array. Bottom: An analytic fit over the 290–1270 keV energy region to a spectrum obtained with statistical α subtraction and in which the event energies were estimated using the total charge collected by the PMT array. In all cases the fitted event rates refer to the total rate of each species, independent of the fit energy window.

statistical subtraction method [12] and a cut-based technique which removes a fraction of the α events with a very small loss of β events. The example spectra shown in Fig. 1 illustrate the stability of our fit procedure; the ${}^8\text{B}$ neutrino and ${}^{214}\text{Pb}$, ${}^{222}\text{Rn}$, and ${}^{218}\text{Po}$ background spectra are small on the scale of the plots and are not shown. The results of these and other fits using different permutations of the fit characteristics and data preparation techniques described above were averaged to obtain the central values reported in Table I; the spread between the results is included in the systematic uncertainty.

The main systematic uncertainties in our measurement of the ${}^7\text{Be}$ interaction rate are listed in Table II. The dominant contributions come from the determination of the fiducial volume, our understanding of the detector energy response, and the variation between the results of

TABLE I. Average fit results [counts/(day · 100 ton)].

${}^7\text{Be}$	$46.0 \pm 1.5(\text{stat})_{-1.6}^{+1.5}(\text{syst})$
${}^{85}\text{Kr}$	$31.2 \pm 1.7(\text{stat}) \pm 4.7(\text{syst})$
${}^{210}\text{Bi}$	$41.0 \pm 1.5(\text{stat}) \pm 2.3(\text{syst})$
${}^{11}\text{C}$	$28.5 \pm 0.2(\text{stat}) \pm 0.7(\text{syst})$

the different fit procedures. We note the significant decrease in the uncertainties associated with the detector energy response and the definition of the fiducial volume, by factors of 4.6 and 2.2, respectively, relative to [12]. This improvement was made possible by several campaigns of detector calibration.

During the first 1.6 years of Borexino operation, the energy and position reconstruction algorithms were tuned, and their performances estimated, using intrinsic activities such as ${}^{14}\text{C}$, ${}^{210}\text{Po}$, and ${}^{11}\text{C}$. The first deployed source calibrations were carried out in 4 campaigns between October 2008 and July 2009. Encapsulated radioactive sources, including ${}^{57}\text{Co}$, ${}^{139}\text{Ce}$, ${}^{203}\text{Hg}$, ${}^{85}\text{Sr}$, ${}^{54}\text{Mn}$, ${}^{65}\text{Zn}$, ${}^{40}\text{K}$, ${}^{60}\text{Co}$, and ${}^{222}\text{Rn}$, were placed inside the scintillator volume using a rod-based source deployment system. Using seven CCD cameras mounted within the PMT array, the positions of the sources could be determined with a precision better than 2 cm. In aggregate, more than 35 live-days of calibration data were recorded with sources at more than 250 positions within the scintillator volume.

The systematic uncertainty in the definition of the fiducial mass was evaluated by comparing the position of the source as measured by the CCD camera system and as reconstructed using the PMT array for a number of runs with sources deployed near the boundary of the fiducial volume. The γ -ray sources were also used to validate and improve both the Monte Carlo calculations (the simulated and observed optical responses agree at the 1.5% level within the fiducial volume) and the detector energy response function used in the analytic fitting procedure. The uncertainties in these tunings and in our understanding of the calibration data were included in the energy scale systematic. A dominant contribution to the latter came from the uncertainty in the scintillator quenching model used to extrapolate the detector optical response from the

γ -ray events used for energy calibration to the single electron events which comprise the majority of the data.

It may be noted in Table I that the ${}^{85}\text{Kr}$ rate has a larger systematic uncertainty than the other fitted rates. This is due to a larger variation in the ${}^{85}\text{Kr}$ rate between the different fit procedures. We typically obtain larger ${}^{85}\text{Kr}$ values when fewer α events are removed from the spectrum before fitting and when the fit range is extended to encompass lower energy events. However, as the ${}^7\text{Be}$ rate is mostly constrained by the spectral region between 550 and 750 keV, the variation in the ${}^{85}\text{Kr}$ rate reflects only weakly in the ${}^7\text{Be}$ rate. We note that the fitted ${}^{85}\text{Kr}$ rate is consistent with an independent measurement of $30.4 \pm 5.3(\text{stat}) \pm 1.3(\text{syst})$ counts/(day · 100 ton) for the ${}^{85}\text{Kr}$ activity obtained using ${}^{85}\text{Kr} - {}^{85\text{m}}\text{Rb}$ delayed coincidences.

Our best value for the interaction rate of 862 keV ${}^7\text{Be}$ solar neutrinos in Borexino is $46.0 \pm 1.5(\text{stat})_{-1.6}^{+1.5}(\text{syst})$ counts/(day · 100 ton). If the neutrinos are assumed to be purely ν_e , this corresponds [17] to an 862 keV ${}^7\text{Be}$ solar neutrino flux [20] of $(2.78 \pm 0.13) \times 10^9 \text{ cm}^{-2} \text{ s}^{-1}$. The corresponding flux prediction from the SSM is $(4.48 \pm 0.31) \times 10^9 \text{ cm}^{-2} \text{ s}^{-1}$, which, if all the neutrinos remained ν_e , would yield an interaction rate of 74.0 ± 5.2 counts/(day · 100 ton) in Borexino; the observed interaction rate is 5.0σ lower. The ratio of the measured to the predicted ν_e -equivalent flux is 0.62 ± 0.05 . Under the assumption that the reduction in the apparent flux is the result of ν_e oscillation to ν_μ or ν_τ (which also undergo electron elastic scattering interactions, but with a cross section about 4.5 times lower than ν_e at this energy), we find $P_{ee} = 0.51 \pm 0.07$ at 862 keV. The improved constraint on the low energy solar P_{ee} is shown in Fig. 2.

We have also performed a global analysis to determine the MSW neutrino mixing parameters in the two-flavor approximation. This included the rate and spectrum information from the other solar neutrino experiments [1–4], the SSM flux predictions [13] (with the exception that the ${}^8\text{B}$ flux was left free), and the current result. We find best fit oscillation parameters of $\tan^2\theta_{12} = 0.468_{-0.030}^{+0.039}$ and $\Delta m_{12}^2 = (5.2_{-0.9}^{+1.5}) \times 10^{-5} \text{ eV}^2$; including the KamLAND reactor antineutrino data [7] these become $\tan^2\theta_{12} = 0.457_{-0.025}^{+0.033}$ and $\Delta m_{12}^2 = (7.50_{-0.24}^{+0.16}) \times 10^{-5} \text{ eV}^2$. The new result slightly improves the precision with which the MSW mixing parameters can be determined.

Alternatively, by assuming MSW-LMA solar neutrino oscillations, the Borexino results can be used to measure the ${}^7\text{Be}$ solar neutrino flux. Using the oscillation parameters from [15], the Borexino result corresponds to a total ${}^7\text{Be}$ neutrino flux $\Phi_{7\text{Be}} = (4.84 \pm 0.24) \times 10^9 \text{ cm}^{-2} \text{ s}^{-1}$ (after oscillation effects, the SSM prediction corresponds to an 862 keV ${}^7\text{Be}$ neutrino interaction rate of 47.5 ± 3.4 counts/(day · 100 ton) in Borexino). The ratio of our measurement to the SSM prediction gives $f_{7\text{Be}} = 0.97 \pm 0.09$.

TABLE II. ${}^7\text{Be}$ systematic uncertainties [%].

Source	[%]
Trigger efficiency and stability	<0.1
Live time	0.04
Scintillator density	0.05
Sacrifice of cuts	0.1
Fiducial volume	$+0.5$ -1.3
Fit methods	2.0
Energy response	2.7
Total systematic error	$+3.4$ -3.6

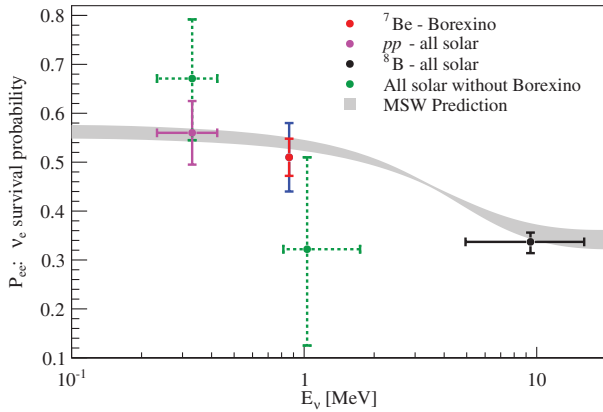


FIG. 2 (color). The global experimental constraints on the low energy solar P_{ee} . For the ${}^7\text{Be}$ point, which shows the current result, the inner (red) error bars show the experimental uncertainty, while the outer (blue) error bars show the total (experimental + SSM) uncertainty. The remaining points were obtained following the procedure in [22], wherein the survival probabilities of the low energy (pp), medium energy, and high energy (${}^8\text{B}$) solar neutrinos are obtained, with minimal model dependence, from a combined analysis of the results of all solar neutrino experiments. To illustrate Borexino's effect on the low energy P_{ee} measurements, the green (dashed) points are calculated without using the Borexino data. The MSW-LMA prediction is also shown for comparison; the band defines the $1 - \sigma$ range of the mixing parameter estimate in [15], which does not include the current result.

Finally, we have used our new result to update the global experimental constraint on the other low energy solar neutrino fluxes by performing a global analysis in which the ${}^8\text{B}$, ${}^7\text{Be}$, CNO, and pp fluxes were left as free parameters, following [21]. Under the luminosity constraint, we find $\Phi_{pp} = (6.06^{+0.02}_{-0.06}) \times 10^{10} \text{ cm}^{-2} \text{ s}^{-1}$ and $\Phi_{\text{CNO}} < 1.3 \times 10^9 \text{ cm}^{-2} \text{ s}^{-1}$ at 95% C.L. Expressed as a fraction of the SSM-predicted fluxes these correspond to $f_{pp} = 1.013^{+0.003}_{-0.010}$ and $f_{\text{CNO}} < 2.5$ at 95% C.L. The latter limits the CNO contribution to the solar luminosity to $< 1.7\%$ (95% C.L.). Both the precision of the pp flux determination and the constraint on the CNO flux are improved by approximately a factor of 2 by our new result.

We have presented a measurement of the interaction rate of 862 keV ${}^7\text{Be}$ solar neutrinos in Borexino with a total uncertainty less than 5%. This precise measurement has also improved our experimental understanding of the other low energy solar neutrinos. The Borexino measurements have allowed us to test and validate the MSW-LMA model in the vacuum oscillation regime and at the lower energy edge of the transition region with unprecedented precision.

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